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NOTES
ON
KINEMATICS OF MACHINERY

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PREPARED FOR THE USE OF THE STUDENTS
IN THE
MECHANICAL ENGINEERING DEPARTMENT

by

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- - -

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KINEMATICS OF MACHINERY

CHARACTERISTICS OF MACHINE MOTION:

PAIRS, LINKS, CHAINS AND MECHANICS.

1. Kinematics is that branch of mechanics that deals with motion; hence kinematics of machinery is the special branch of kinematics that has to do with the motions of various parts of the machine. The problems that arise in kinematics of machinery are of two kinds:- (1) having a machine or mechanism, to determine the relative or absolute motions of its parts; (2) to discover the mechanism or arrangement of parts by means of which any required motion may be obtained. Problems of the second kind are those that frequently confront the designer of machinery.

The characteristics of the motion of a machine part as distinguished from the motion of a free body is that the motion of the former is completely constrained. The rigid pieces that together form a machine or mechanism are so connected that when one changes its position others must change their positions also, and these changes must follow some law precisely defined by their nature of mechanism. Or to put the statement in other words, every point in every machine part moves in a fixed predetermined path.

2. Pairs. It is evident that each rigid pieco of a mechanism must have contact with one or more of the other pieces of the mechanism; for if such were not the case, the piece in question would be a free body and its motion would not be constrained. The connection between the pieces constituto what we call pairs. The

connection between two pieces may be:- (1) such that the relative motion of the two pieces is fixed by the form of connection; (2) such that the relative motion is not thus fixed but is determined by the connection of the pieces with still other bodies. In the first case the pair is complete or closed; in the second case the pair is said to be incomplete. For example, the two legs of a pair of dividers are connected by a joint such that the only possible motion of one leg relative to the other is rotation about the axis of the joint; the connection between the cross-head and guides of a steam engine is of such a nature that the only possible motion of the cross-head is a sliding or translation. On the other hand, the connection between two gear wheels is of the second kind. The mere contact between the teeth is not sufficient to constrain the relative motion of the wheels, and it is only when the wheels are connected to some third body that constrained motion is possible.

3. Lower Pairs. Let us investigate the forms of connection to be employed when it is desired to constrain the relative motion of the two pieces connected. Suppose we take first a motion of rotation. If upon one body we construct a cylindrical projection and in the other make a cylindrical recess or hole of the same diameter and insert the projection in the recess, then evidently the relative motion of the two bodies is a rotation about the axis of the cylinder, and provided we prevent the bodies from separating, no other relative motion is possible. This kind of pair, the pin and eye, or journal and bearing, is called the turning pair. If the desired relative motion is a translation, we provide each of the pieces with parallel plane surfaces; thus we can cut a rectangular slot in one

piece and provide the other with a rectangular projection having the same width as the slot. A pair of this kind is called a sliding pair; a familiar example is the pair between the cross-head and guides of the steamengine. A third pair called the screw pair is exemplified in the ordinary screw and nut, the relative motion of the pieces in this case being a combined rotation and translation.

The pairs just described are called lower pairs. They have two characteristics that render them peculiarly useful in machine construction. (1) They are easily constructed; the cylinder and screw by the lathe, and the plane surface of the slide by the planer or shaper. (2) The contact between the elements of each pair is a surface contact, a desirable feature from the standpoint of durability. A kinematic property of these pairs which may be used as a definition of lower pairs is the following:- Let the two bodies connected by the pairs be denoted by a and b, and let P be the point on a and P' a point on b; when b is at rest and a moves, the point P will describe a curve m; likewise when a is at rest and b moves, the point P' will describe a curve n; if these curves are similar, the pair is a lower one. It is this property rather than the fact of surface contact that defines the lower pair; for example, a cylindrical shaft rotating in a square bearing has only line contact with the bearing; but a point on either shaft or bearing describes a circle when the other part is held stationary, and therefore, we consider the pair a lower pair, notwithstanding the character of the contact.

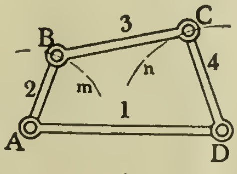
4. Higher Pairs. Pairs for which the kinematic property just stated does not hold are called higher pairs. The contact in these

pairs is always line contact, and in general these pairs are incomplete, though closed or complete higher pairs are possible. Gears and cams furnish examples of higher pairs.

5. Pairing Elements. The geometrical forms given to or placed upon the two bodies so that they may be connected by a pair are called pairing elements. Thus the journal or pin on one body and the bearing or eye on the other are pairing elements. In the case of the steam-engine cross-head, the plane surface that comes in contact with the guide is one element and the cylindrical pin that fits the eye in the connecting rod is a second element.

6. Kinematic Links and Chains. A body provided with two or more pairing elements so that it may be connected to at least two other bodies is called a kinematic link or simply a link. A link carrying two elements we call a binary link; one with three elements, a ternary link, and so on.

Having several links provided with suitable elements, we may connect them by pairing the elements. Thus, if we have four links a, b, c, d each with two elements, we may pair one element of a with one of b's; the other element of b with one of c's, and the other element of c with one of d's and finally the other element of d with that element of a still unpaired. The series of links thus arranged - connected so that each element is paired with a partner and none left single - constitute a kinematic chain.



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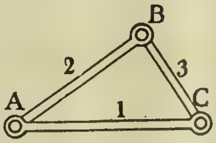

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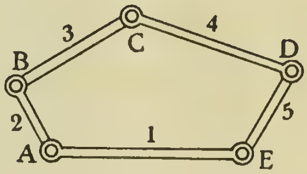

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7. *Constraint*. In Figs. 1, 2 and 3 are shown chains composed respectively of 4, 3 and 5 binary links connected by turning pairs. It is quite evident that in each of the three cases the relative motion of any two adjacent links is completely constrained and consist sists of a rotation about the axis of the apri; thus in Fig. 1, the motion of 2 relative to 1 is a rotation about the axis of the pair A, and that of 4 relative to 1 is a rotation about the pair D. We now investigate the motion of non-adjacent links. In Fig. 1, suppose the link 1 to be held fast, say to the paper, and suppose the link 2 to be put in motion. The only possible motion of 2 is a rotation about the axis of A; hence the pair B must move in a circular arc m. The motion of 2 will cause links 3 and 4 to move also, and since the pair D is stationary, 4 must rotate about the axis of D and the pair C must move in ancircular arc n. Now the centers of the pairs B and C are points of the link 3, and since the motion of these two points are fixed, the motion of 3 relative to the paper is completely determined; that is, the motion of 3 relative to the non-adjacent link 1 is constrained. In a similar manner it can be shown that the motion of 2 relative to 4 is constrained. A chain which has this property - i.e. each link has constrained motion relative to every other link - is a *constrained chain*.

Let us now examine the 5-link chain shown in Fig. 3. If, as before, we consider link 1 fixed to the paper, the adjacent links 2 and 5 will have constrained motion, rotations about the axes of the pairs A and E, respectively. The axes of the pairs B and D will therefore move in arcs of circles with A and E as centers; hence

we know the motion of one point B of a body 3 and the motion of one point D of link 4. But the motion of one point of a body is not sufficient to determine the motion of the body; hence the motions of the links 3 and 4 are indeterminate, that is, unconstrained relative to the non-adjacent link 1. A chain of this character is an unconstrained chain.

In the chain shown in Fig. 2 it is readily seen that the links as connected can have no relative motion. Thus, if 1 is held fast the axis of the pair B must rotate about A as a center, if we consider B a point of link 2; but B is also a point of link 3 and as such must rotate about O as a center; hence since B cannot move in two separate paths at the same time it cannot move at all. A chain in which there can be no relative motion between the links is called a locked chain. It is evident that a locked chain cannot be used in machine construction for the parts of the machine are supposed to move relatively to one another; they are, however, used freely in rigid constructions such as bridge and roof trusses.

The unconstrained chain is entirely useless. It cannot be used in machine construction for the links being unconstrained could move in any one of an infinite number of ways depending upon the forces that might be acting at that particular instant. The constrained chain is the only one that is useful from our point of view.

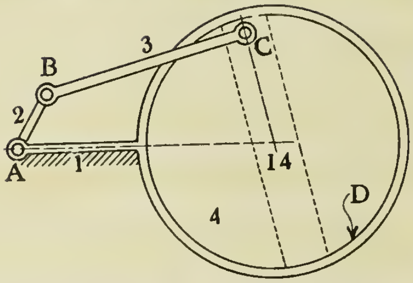
8. Mechanisms. The step from the constrained chain to the mechanism is simple. Suppose one of the links of the chain to be fixed or held stationary relative to the earth or some other standard, then since each link has a definite motion relative to the fixed link

it will have a definite constrained motion relative to this standard. The chain with one of its links fixed constitutes a *mechanism*.

Any one of the links of a constrained chain may be chosen as the fixed link; hence a chain has as many mechanisms as it has links.

9. *Expansion of Pairing Elements*. The relative motions of the links of a mechanism depend only upon the kinds of pairs employed and the relative position of these pairs. The size or form of the link itself is immaterial and the size of the pairing elements has no influence on the motion. In Fig. 1, the links are represented as straight bars carrying small pins or eyes at their ends; in the actual machine, however, the links may have very different forms, in particular the fixed link or frame of the machine, which may contain tons of cast iron. No matter what form link 1 may have, if it carries the pairing elements A and B in the proper positions, the relative motions of the links will be the same. Evidently also if the axis of the turning pair A be kept in the same position, a change in the diameter of the pin and eye can have no effect upon the motion of 2 relative to 1.

Figs. 4 to 9, which are taken from Kennedy's Mechanics of Machinery, show how a mechanism may be disguised by enlargement of pairing elements. The chain shown in Fig. 4 is precisely the same as that shown in Fig. 1 except that the pair A has been enlarged to such an extent that link 2 has become a disc whose circumference is one of its pairing elements. In Fig. 5 the same link 2 is again a circular disc but in this case the pair B has been expanded. The link 2 in this case is an accentric; hence, comparing Fig. 5 with Fig. 1, we



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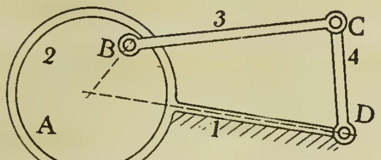
see that an eccentric is equivalent to a crank whose length is the distance between the axes of the pairs. In Fig. 6 the pair D is



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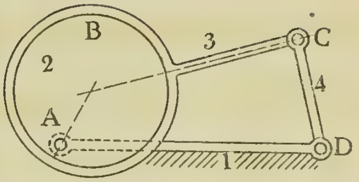

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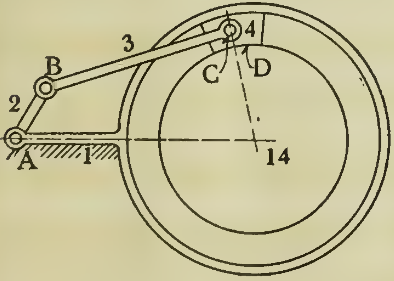
enlarged. Evidently the motion of the link 4 will be equally well constrained if the sides of the circular disc are cut away leaving the zone shown by the dotted lines. Another modification is shown in Fig. 7, where the link 4 is given the form of an annular ring instead of a disc. The next step is to cut away all of the ring except a small block containing the pair C. Finally since by the rotation of crank 2, the pair C reciprocates in a limited circular path, we may as well shorten the circular slot in which 4 moves to the actual length required, as shown in Fig. 8. By these modifications we have in no way changed the character of the motion. In each case (Figs. 1, 6, 7, 8) the pair C is constrained to move in the same path, a circular arc with the axis (1, 4) of the pair D as a center.



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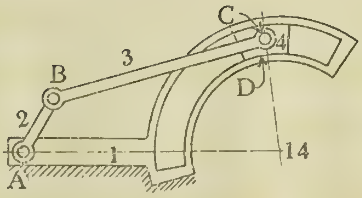

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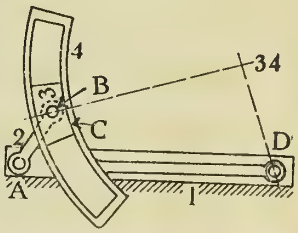

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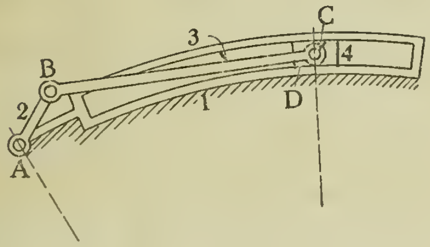
By a series of modifications similar to those just described, the arrangement shown in Fig. 9 may be derived from that shown in Fig. 1. The link 4 is a curved slot in which the block reciprocates. The point (3, 4), the center of curvature of the slot, is the axis of the pair C.

10. The Relation Between Turning and Sliding Pairs. We have seen that the curved slot and block, as in Fig. 8, are the elements of a turning pair whose axis passes through the center of curvature of the slot. From a kinematic point of view the length of a binary link is the distance between the axes of its parts:- Thus in Fig. 8, the actual length of link 4 is (1, 4) C, and that of link 1 is (1, 4) A. By using the curved block and slot we are enabled to get long links without increasing the size of the mechanism. Thus, if we give the slot less curvature, as shown in Fig. 10, the center of curvature (1, 4) is made to recede, and the kinematic lengths of the links 1 and 4 are increased. We may imagine the slot to be flattened more and more until its sides are straight, as shown in Fig. 11; the center of curvature in this case is at an infinite distance and the links 1 and 4 are infinitely long. From these considerations it appears that the sliding pair, as D, Fig. 11, is really a turning pair whose axis is at an infinite distance in a direction at right angles to the direction of the sliding.



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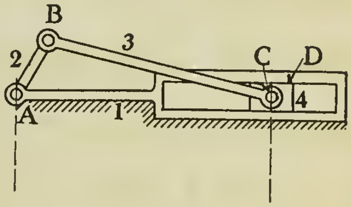

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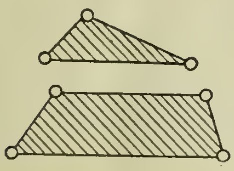
The mechanism shown in Fig. 11 with three turning pairs and one sliding pair is called the *slider-crank* of steam-engine mechanism; it is the mechanism used in the ordinary steam-engine.

11. *Formation of Constrained Chains.* Having given a series of links each with two or more pairing elements we may combine these links to form chains. As we have seen, the chains thus formed may be constrained, unconstrained or locked. As constrained chains are the only ones admissable, the problems that we are to solve are (1) to join a series of links so as to form a constrained chain, and (2) to derive the conditions that must be fulfilled if the chain is constrained.

A ternary link may be represented by a simple triangle with small circles at the vertices to denote the pairing elements; likewise, a quaternary link by a four-sided figure, etc. See Fig. 12. To build up a constrained chain from binary and ternary links, let us start with the four-link chain. In Fig. 13, we start with the links 1, 2, 3, 4 giving us a four-link chain with A, B, C and D pairs. Suppose we add an element E to link 3 and an element F to link 4, thus making links 3 and 4 ternary: and let us join E and F with two binary links 5 and 6 having a common pair G. The result is a six-link chain with two ternary and four binary links. The



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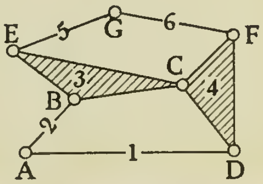

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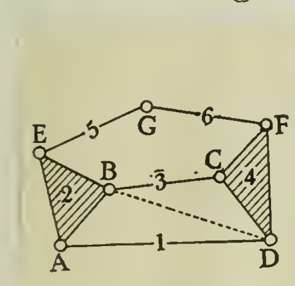
chain is constrained, for E and F have definite motions in known paths and this fact imposes upon G a determinate motion. If we connect E and F with one link, we lock the chain, and if we use more than two links the chain is unconstrained. This chain is known as the Watt chain because with link 3 fixed it gives the mechanism of the Watt beam engine.

If we add the element E to link 2, making the non-adjacent links 2 and 4 the ternary links, we obtain a second six-link constrained chain shown in Fig. 14. This is known as the Stephenson chain.

In a ternary link two of the elements may coincide. Suppose that the elements D and E of link 3, Fig. 3, thus fall together; the resulting chain is that shown in Fig. 15. This chain is also obtained by making B and E of the link 2, Fig. 14, coincident. We may further make the elements F and D of link 4 coincident, thus obtaining the chain shown in Fig. 16, in which the links are all binary. It is to be noted that neither F nor D may be made to coincide with C, for in that case three of the links would form a locked triangle and the chain would in reality have only four links.



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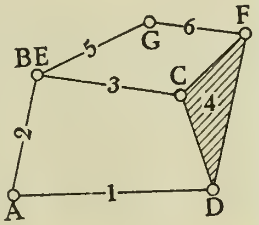

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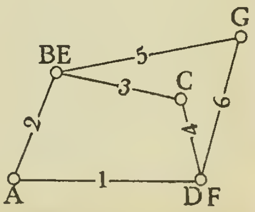

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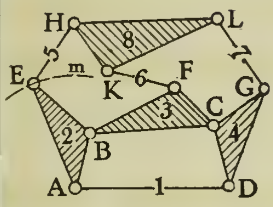
When a joint as B, E or D, F, Fig. 16, is made up of three elements, it is called a ternary joint; one made up of four elements

is called a quaternary joint, and so on. The ordinary joint of two elements is a binary joint. Evidently a ternary joint is equivalent to two binary joints; a quaternary joint to three binary joints; and a joint of i elements to i-1 binary joints. Starting again with the 4-link chain, suppose all three links, 2, 3 and 4 to be made ternary by the addition of the elements E, F and G. At these points we connect three binary links 5, 6 and 7 and pair these with a fourth ternary link 8. The result is an 8-link chain with 4 binary and 4 ternary links. That this chain is constrained is established by the following reasoning. Consider the link 1 fixed and let the pairs at E be separated. We will denote the element of the pair E that belongs to 2 by E_2 and the element that belongs to 5 by E_5. If the link 2 be moved, E_2 must move in a curve m and the joints F and G will likewise move in definite paths. Suppose E_2, F, and G moved to some position; then keeping F and G at rest and moving H, K and L will be made to describe definite paths, and as a consequence H will be forced to move in a fixed path n. At the same time E_5 may be made to move in the path m, and therefore may be brought into coincidence with E_2. These two operations may be repeated indefinitely; E_2, F and G are first moved, then held fixed while K, L, and H are moved, and E_5 is made to coincide with E_2. But these successive motions may be made as small as we please and may take place at the same time instead of in succession. In this case E_2 and E_5 will remain together and may be joined to form a pair. Now since for every position of E, F, and G, H, K, and L,



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have each definite positions when E_5 is brought to E_2, it follows that when E_5 and E_2 remain together H, K, and L move in definite paths. This fact proves the constrainment of the chain.

Other 8-link constrained chains are shown in Figs. 18 and 19. In the chain Fig. 18, two 4-link chains with a common link are joined by the link 8. If 1 is held fixed the points E and F are evidently constrained to move in definite paths; hence if E and F are joined by the link 8, that link must have a definite motion relative to link 1.

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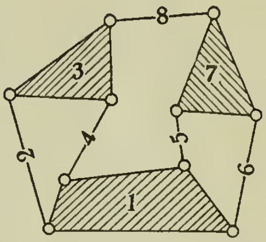

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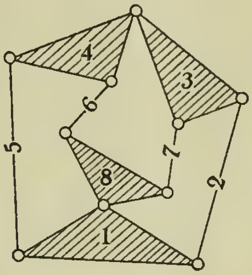

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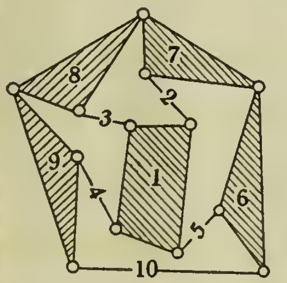
Figs. 20 and 21 show chains with 10 and 12 links, respectively. The constrainment of the 12-link chain is easily proved, there being no less than three 4-link cells. It is not easy however to determine whether the chains shown in Figs. 19 and 20 are constrained, as all the cells are bounded by five or more links. In general, it is difficult or impossible to determine by inspection whether a



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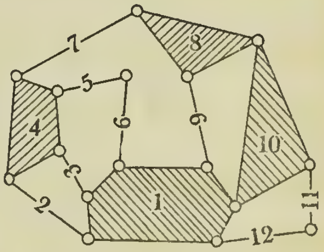

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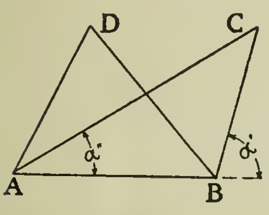
chain is constrained or not when the chain has more than six links. There is, however, a simple relation between the number of links and the number of points in a constrained chain that may be used as a criterion of constraint. This relation is deduced in the next paragraph.

12. Criterion of Constraint. In order that several points A, B, C, D, etc., may occupy always the same relative position a certain number of conditions are necessary. With two points A and B there is one condition required, namely that the points shall be at the same distance apart or that the length of the joining line A B shall be constant. If a third point C is added to the system, two more conditions are required to fix C relatively to A and B. These may be the lengths of the joining lines A D and B C or the angles a' and a", which these lines make with A B. Three points are therefore fixed by three conditions. Similar reasoning shows that a fourth point D requires two more conditions, and each additional point two additional conditions. In general, to fix the relative positions of n points 2n-3 independent conditions are necessary.



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In a chain the joints are the characteristic points whose relative positions are to be investigated; and the necessary conditions are furnished by the links. Consider the chain shown in Fig. 14. If we connect the joints B and D by a rigid link, as shown by the dash line, we know that the chain will be locked. Thus the addition of one condition to a constrained chain locks it, and conversely the

removal of one condition in the case of a locked chain, gives a constrained chain, a chain with ono degree of freedom.

Let J = number of joints in chain;

C = conditions furnished by links;

We have seen that 2J-3 conditions just locks the chain; hence 2J-4 conditions leaves the chain constrained. Expressed in algebraic languago,

if $C = 2J-3$, the chain is locked;

$C \geqq 2J-4$, the chain is constrained;

$C \leqq 2J-4$, the chain is unconstrained.

Each link of the chain furnishes a number of conditions depending on the number of its pairing elements. A binary link gives one condition, the distance between the joints connected by it; a ternary link gives three conditions, the three required to fix the relative positions of its threo elements; a quaternary link contributes the five independent conditions that fix its four elements. In general, a link with k pairing elements oontributes 2K-3 independent conditions. Let N = total number of links in chain;

J = total number of joints in chain;

E = total number of pairing elements;

n_2, n_3, n_4, etc. = respectively, the number of links of 2,3,4, etc. elements m_2, m_3, m_4, etc. = respectively number of binary, ternary, etc. joints;

C = number of conditions furnished by links.

Then $E = 2n_2 + 3n_3 + 4n_4 + \ldots\ldots\ldots\ldots$ (a)

also $E = 2m_2 + 3m_3 + 4m_4 + \ldots\ldots\ldots\ldots$ (b)

$N = n_2 + n_3 + n_4 + \ldots\ldots\ldots\ldots$ (c)

$$J = m_2 + m_3 + m_4 + \dots\dots\dots \quad (d)$$

$$C = n_2 + 3m_3 + 5n_4 + 7n_5 + \dots\dots \quad (e)$$

If this chain is constrained,

$$C = 2J-4;$$

hence $n_2 + 3n_3 + 5n_4 + 7n_5 + \dots\dots\dots\dots = 2J-4$ (1)

This equation is the criterion of the constrainment of the chain.

Adding equations (c) and (1), we obtain

$$2n_2 + 4n_3 + 6n_4 + 8n_5 + \dots\dots\dots\dots = 2J-4+N \quad (f)$$

Subtracting (c) from (a)

$$n_2 + 2n_3 + 3n_4 + 4n_5 + \dots\dots\dots\dots = E-N \quad (g)$$

Comparing (f) with (g), we readily obtain

$$2(E-N) = 2J-4+N$$

or $$E-J = \frac{3}{2}N - 2 \quad (h)$$

If the chain has only binary joints,

$$E = 2m_2 = 2J$$

and (h) reduces to

$$J = \frac{3}{2}N-2. \quad (2)$$

Equation (2) is the form in which the criterion is most generally used. Even when a chain has ternary and quaternary joints, we may consider each ternary joint replaced by two binary joints, each quaternary joint by three binary joints; and so on, and use the form (2). From the form of (2) it appears that a chain with only lower pairs cannot in general have an odd number of links.

As an exercise, let the student apply both criterions to each of the chains shown in Figs. 13 to 21.

13. Constrainment of Chains With Higher Pairs. The term joint is restricted to the connection of two links by a complete

or closed pair. The mere point of contact which ordinarily occurs in higher pairing is not a joint in this sense. It chains with higher pairs, therefore, there may be links with only one joint; gear wheels and cams are nearly always links of this character. In conformity with the notation of the preceding paragraph, the number of such links is denoted by n_1. Now it has been shown that a link with k joints gives 2k-3 conditions; following this rule each link with one joint gives $2 \times 1 - 3 = -1$ conditions. The interpretation of this statement is simple:- such a link with its one joint furnishes no conditions at all, but instead requires one condition, viz., that it shall be in contact with or have higher pairing with some other link.

In addition to the conditions furnished by the links carrying two or more joints, each higher pair furnished one condition, the condition that some pair of links have contact. If H denotes the number of higher pairs, then these higher pairs furnish H conditions. The total number of conditions furnished is therefore

$$C = -n_1 + n_2 + 3n_3 + 5n_4 + \dots\dots\dots\dots + H.$$

As before, the number of conditions necessary to render the chain constrained is $C = 2J-4$; hence

$$-n_1 + n_2 + 3n_3 + 5n_4 + \dots\dots\dots\dots + H = 2J-4 \quad (3)$$

Equation (3) is the most general form of the criterion for chains with higher pairs. A form analagous to equation (2) may be derived as follows: $-E = n_1 + 2n_2 + 3n_3 + 4n_4 + \dots\dots;$

$$N = n_1 + n_2 + n_3 + n_4 + \dots\dots;$$

$$2J - 4 - H = -n_1 + n_2 + 3n_3 + 5n_4 + 7n_5 + \dots;$$

Subtracting and adding, $E - N = n_2 + 2n_3 + 3n_4 + 4n_5 + \dots;$

$$2J - 4 - H + N = 2n_2 + 4n_3 + 6n_4 + 8n_5 + \dots;$$

Therefore

$$2(E - N) = 2J - 4 - H + N$$

or

$$E - J = \frac{1}{2}(3n - 4 - H)$$

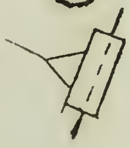
When all the joints are binary, $E = 2J$; hence for this case

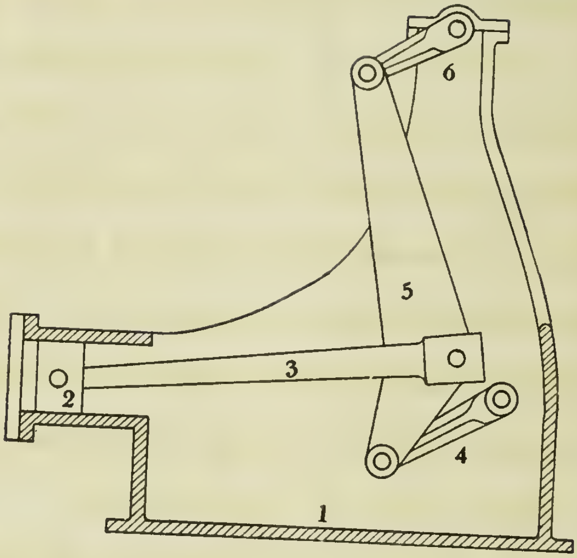
$$J = \frac{1}{2}(3n - 4 - H).$$

In the rarer cases in which higher pairs are closed pairs, they should be counted among the joints J rather than among the higher pairs H.

14. Analysis of Mechanisms. The process of analyzing a mechanism is the converse of the process of building up constrained chains considered in the last section. We have given a mechanism - perhaps a whole machine - with links of various forms, depending upon considerations of strength or stiffness. We identify the pairs and classify them. The analysis of a mechanism consists of the following steps:

(a) Identify each link and give it a number; give the fixed link the number 1.

(b) Represent the links by their conventional forms; a binary link by a straight line with elements at its ends; a ternary link by a triangle with elements at its vertices, and so on. The elements are represented as follows:- an element of a turning pair, thus, ⊙ ; an element of a sliding pair by a simple block, thus,  ; a higher pair by a small triangle, thus, △ .



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(c) Number each element, using the two numbers of the two links joined. Thus, take the pair that joins links 2 and 3; this pair will be denoted by the symbol (2 3) and the elements on both link 2 and link 3 will be given the same symbol.

(d) Having each link in its simplest form, (straight line, triangle, etc.) and with its pairing elements numbered, join the links to form a chain, as shown in Fig. 17 and Fig. 19. Elements having the same number are joined.

(e) Test the constrainment of the skeleton chain thus formed by the equation for constraint. If the equation is not satisfied study the character of the motions and discover a reason for the fact.

(f) The skeleton, if complicated, may be arranged in more symmetrical form by changing dimensions only.

An example will show the process more clearly. The mechanism of the atkinson gas-engine is shown in Fig. 23, and may be described as follows:- The piston 2 reciprocates within a cylinder attached to the fixed link 1 and gives motion to a connecting rod 3. The other end of 3 joins a link 5, which link also pairs with links 4 and 6. Note that 3 pairs with 5, not with 4.

Taking the link 1 - the bed of the enging - we note that it pairs with the links 2, 4 and 6, and has therefore three pairing elements.

In Fig. 24 the link 1 is represented by the triangle 1 with the

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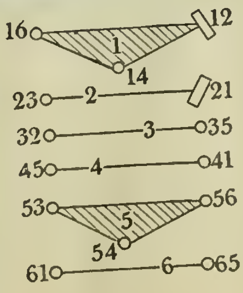

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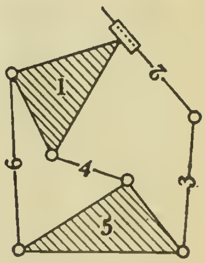

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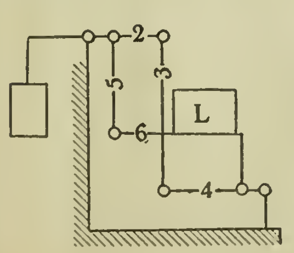
elements (12), (14), and (16) at the vertices. Since the pair (12) is a sliding pair the element (12) on link 1 is represented by the block. The piston 2 is evidently a binary link having a sliding pair with 1 and a turning pair with 3; hence it is represented as shown in Fig. 24. The links, 3, 4 and 6 are binary with turning pairs are therefore represented as shown. The link 5 has three turning pairs and is therefore represented by the triangle 5, Fig. 24. Now joining the six links of Fig. 24 so that the proper pairing eleemnts are connected we obtain the chain shown in Fig. 25. There are 6 links and 7 joints, and all joints are binary; hence applying the criterion $J = \frac{3}{2} N - 2$, $7 = \frac{3}{2} \times 6 - 2$, and the chain is constrained.

When the mechanism is shown in its skeleton form as in Fig. 27, it is readily seen to be identical with the Stephenson mechanism shown in Fig. 14.

15. *Lower Pairs with Multiple Contact.* It frequently occurs in machine construction that a single lower pair may have several contact surfaces. Thus the shaft of an engine runs in two bearings, and both the cross-head and piston are provided with sliding pairs. A single sliding pair may have any number of contact surfaces provided these surfaces are parallel, and a single turning pair may



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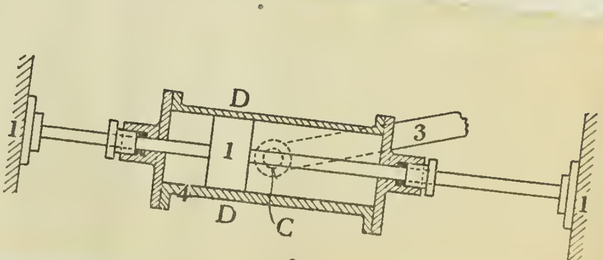

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have any number of journals and bearings provided they all have the same axis. In the analysis of mechanisms, the student must guard against the common error of counting each contact surface a pair.

16. Inversion of Mechanisms. A mechanism is derived from a chain by fixing one of the links of the chain; hence since any link may be chosen as the fixed one, as many mechanisms may be derived as the chain has links. Consider the chain shown in Fig. 14. If the link 6 is fixed the resulting mechanism is that of the Stephenson link, and of the Wenzer needle-bar motion; if instead, link 2 (or link 4) is held fixed, the resulting mechanism is that of the Atkinson gas-engine, the Crosby indicator and the Blake stone crusher; finally with link 1 fixed the mechanism is that of the weighing-scale shown in Fig. 26. The process of deriving one mechanism from another by a change in the fixed link of the chain is called the inversion of the chain.

Problem. The slider-crank chain shown in Fig. 11 becomes the mechanism of the ordinary steam-engine when the link 1 is fixed. Transform this mechanism into three other steam-engines by successively fixing links 3, 2 and 4. Make the length of link 2 - 1" and that of link 3 - 3-1/2" in each case. Be sure that in each case

link 1 joins sliding pair D to turning pair A
link 2 joins turning pair A to turning pair B
link 3 joins turning pair B to turning pair C
link 4 joins turning pair C to sliding pair D

Inversion of Pairs. In the solution of this problem it will be desirable in one case at least to interchange the elements of one or more pairs. The exchange of the solid and hollow elements of

a lower pair does not affect the relative motion of the links connected by the pair and is always permissable. Take for example, the pair B, Fig. 11; usually the crank 2 carries a pin which fits into an eye in the connecting rod 3; sometimes, however, the rod 3 carries the pin and the crank 2 the eye, this construction being common in the cheaper class of machinery. In the case of the sliding pair D, the link 1 usually carries the hollow enclosing element, i.e., the cylinder and guides; this pair may likewise be inverted by making the link 4 carry the enclosing element, as shown in Fig. 27. The inversion is inexpedient when 1 is the fixed link, but is useful when 4 is taken as the fixed link, as it makes the lighter element the moving part.

Conditions to be Observed. (a) Provide each engine with inlet and outlet for steam but do not design valve gear; (b) provide each engine with means of communicating motion to a shop shaft but do not add extra links (an extra belt may be used when 4 is the fixed link); (c) see that no link interfers with the proper motion of any other link; (d) see that the parts are given such dimensions that the required motion is possible; (e) avoid arrangements that give heavy masses reciprocating motions, and balance rotating parts as far as possible. First make freehand sketches in note books and submit before beginning finished drawing. Make the drawing neat and workmanlike, but do not waste time drawing small details such as keys, bolts, etc.

Work independently in solving these problems.

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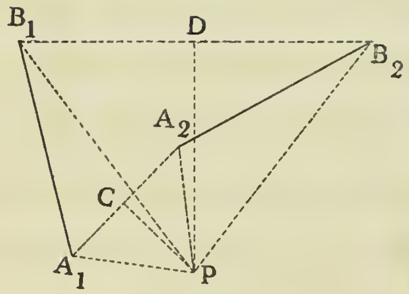

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II. PLANE MOTION OF RIGID SYSTEMS.

17. If a body or rigid system has a motion parallel to a fixed plane, the motion is completely represented by the motion on any plane section of the body in its plane; hence in dealing with cases of plane motion we shall consider the body or system as represented by a plane figure lying in the plane of the paper.

The position of the plane figure is determined by the position of any two points or by the position of any line lying in the plane of the figure; hence the motion of the figure in its plane is completely determined by the motions of two of its points or by the motion of a straight line lying in it.

18. Displacement Produced by a Rotation. Any displacement of a plane system in its plane may be produced by a rotation about some point in the plane. Let A_1B_1 and A_2B_2 be the initial and final positions of some line of the system. At the middle points C and D of A_1A_2 and B_1B_2 we erect perpendiculars which intersect in the point P. Since the triangles PA_1B_1 and PA_2B_2 are equal, it appears that a rotation about the point P will bring one triangle into coincidence with the other and make A_1B_1 coincide with A_2B_2.

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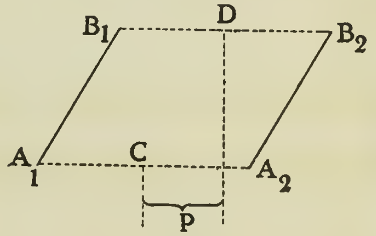

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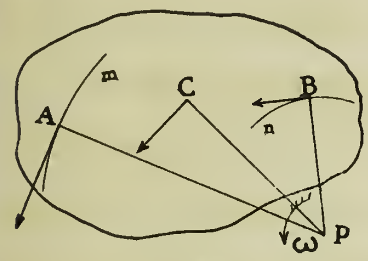
In case the displacements A_1A_2 and B_1B_2 are parallel and equal,

as shown in Fig. 29, the center of rotation falls at an infinite distance and the rotation reduces to a translation.

19. Instantaneous Center, or Pole. Suppose that the displacements A_1A_2 and B_1B_2 are taken indefinitely small. The limiting directions of these displacements will be the directions of the paths of the points A_1 and B_1, and the perpendiculars CP and DP will, when A_1A_2 and B_1B_2 are indefinitely small, become normals to the paths of A_1 and B_1 at the points A_1 and B_1 respectively. Thus in Fig. 30 the point A of the plane system is moving in a path m and the point b in a path n. An infinitesimal displacement of the system causes infinitesimal displacements of the points A and B along the paths m and n, and the perpendiculars to these displacements must be normals to m and n at the points A and B. The intersection P of these normals is the center of rotation for the infinitesimal displacement, or it is the point about which the system is rotating at the instant in question; it is therefore called the instantaneous center of the system. The shorter word pole is used in German texts, but we will retain the longer term and for the sake of brevity use the abbreviation I. C.



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The instantaneous rotation of the system as a rotation about P is the same as it would be if P were a point in a permanent fixed axis passing through the rotating body. Let the instantaneous angular velocity of the rotation about P be denoted by ω;- then treating P as a point in a fixed axis, the linear velocity of the point A is PA-ω, that of B is PB-ω. In general, the linear velocity

[illegible]

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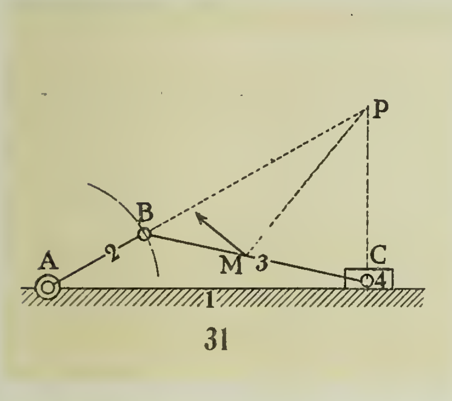
of any point in the moving system is obtained by multiplying the angular velocity (ω) by the distance between the point and the I. C. The direction of the velocity of any point is perpendicular to the instantaneous radius joining the point to P; thus in Fig. 30 the direction of the velocity of C is perpendicular to PC.

As an illustration of these principles let us study the motion of the connecting-rod of the steam-engine mechanism, Fig. 31. The link 1 is considered fixed relative to the paper but the links 2, 3 and 4 are moving systems. The motion of the crank 2 is a rotation about the fixed center A and the motion of the block 4 is a pure translation in the direction AC; the motion of the rod 3 is more complicated.



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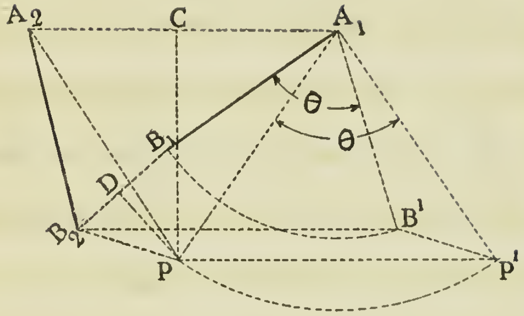
Considering the rod as a plane system, we know that the motion of the system is determined by the motion of any two of its points. The joints B and C are two points belonging to the rod whose motions are known. The path of B is a circle with A as a center and that of C is the straight line AC. The normal to the path of B evidently has the direction AB; hence prolonging AB and erecting a perpendicular at C to AC, we obtain the I. C. of the moving system 3 at the intersection of P of these lines. The I. C. being known, we can determine the velocity of all points of the system; thus the point M is moving in a direction at right angles to MP and the magnitude of the velocity of M is given by the proportion

lin. vel. B: lin. vel. C : lin. vel. M = PB : PC : PM



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20. Combined Rotation and Translation. A displacement of any plane system may, as we have seen, be accomplished by a single rotation about some one point of the system. There are, however, other ways of making the displacement; we may find that we can give a system any required displacement in its plane:- (1) by a combined rotation and translation; (2) by two successive rotations; (3) in general, by any number of successive rotations and translations.

Let A_1B_1 and A_2B_2, Fig. 32, be the initial and final positions of some line in the moving system. As in Fig. 28, we readily find the pole P, a rotation about which will give the system the required displacement. Let Θ denote the angle between the directions of A, B, and A_2B_2; then Θ must be the angular displacement about the pole P. Now suppose we take the point A_1 as the center of rotation and give the system a counter-clockwise rotation of such magnitude that A_1B_1 will take the positinn A_1B' parallel to A_2B_2. Evidently the angle B_1A_1B' = Θ, the angular displacement of the system. The rotation carried P to P' (angle PA_1P' = Θ), and the triangles $A_1B'P'$ and $A_2B_2P_1$, which by construction are equal, have their sides parallel. If therefore we give the triangle $A_1B'P'$ a translation equal to P'P, A_1 will fall at A_2, B' will fall at B_2, and P' will fall at P, its original position. The rotation about A_1 followed by the translation has effected the desired displacement of the system, and as it should, has left the position of the pole unchanged.

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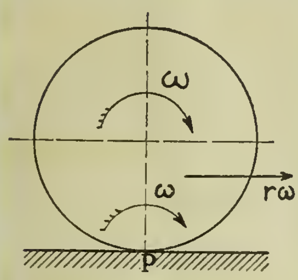
Let p denote the distance A_1P between the point A_1 about which

the rotation occurs and the pole P, and let s denote the length PP'; then from the triangle PA_1P' :- $s = 2p \sin \frac{\Theta}{2}$.
We have then the following:- *A rotation of the plane system through an angle Θ about a point P is equivalent to a rotation of angle Θ about another point of the system at a distance p from P and a translation of magnitude $2p \sin \frac{\Theta}{2}$.*

Suppose we consider the displacement indefinitely small. Then it may be effected by either a rotation about P of angle $d\Theta$ or by a rotation about some other point, say L_1 of the same angle $d\Theta$ and a translation of magnitude $2p \sin \frac{\Theta}{2} = pd\Theta$, where as before, p is the distance between A_1 and P. Referring to Fig. 32, we see that the direction of the displacement P'P makes an angle $\frac{\pi}{2} - \frac{\Theta}{2}$ with A_1P; hence when the displacement is taken indefinitely small and the angle Θ approaches zero, the limiting direction of the translation is perpendicular to A_1P. Hence the followgin statement:- *An infinitesimal rotation of a plane system about a point P of the system may be replaced by an infinitesimal rotation about any other point and a simultaneous translation perpendicular to the line joining the points.*

If we consider each of the rotations accomplished in the infinitesimal interval of time dt, then since $\frac{d\Theta}{dt} = \omega$, the magnitude of the angular velocity of the system, the preceding statement may be given the following form, -- *An angular velocity of a rigid plane system about any point can be resolved into an equal angular velocity about another point at a distance p from the first and a velocity of translation $v = p\omega$ at right angles to the joining line of the points.*

Illustrations of this important principle are shown in Figs. 33 and 34. A disc or wheel of radius r rolls on a level surface. The point of contact P is evidently the center about which the wheel is rotating at any instant. The rotation about P of angular velocity ω can be resolved into a rotation of equal angular velocity ω about O, the center of the wheel and a translation of linear velocity $v = r\omega$ at right angles to OP, that is, parallel to surface on which the wheel is rolling. In other words, the actual motion of the wheel is the same as if it really rotated about its center O, the center in the meantime moving parallel to the track with the velocity $r\omega$.



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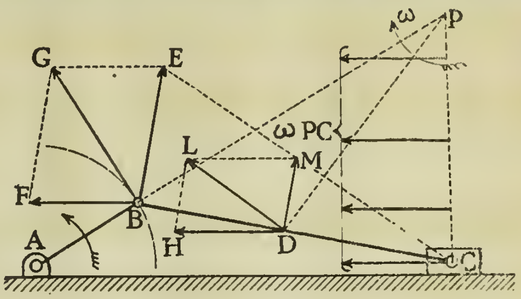

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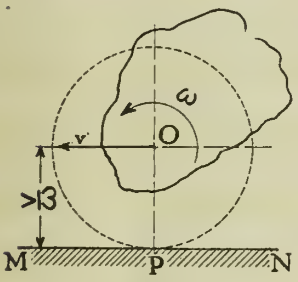
In Fig. 34 the motion of the connecting-rod 3 is a rotation about the center P. This motion may be represented by a rotation of equal angular velocity ω about the end C of the rod combined with a translation of linear velocity ω PC at right angles to PC. Evidently the linear velocity of any point of the rod can be resolved into two components, one the velocity due to the rotation about C, the other the velocity of translation ω PC. Thus the velocity of the point B of the rod is perpendicular to PB, has the magnitude ω PB and may be represented by the vector BG. This velocity is resolvable into the components BE $= \omega$ BC and BF $= \omega$PC, the first due to the

rotation about C and perpendicular to BC, the second the velocity of translation common to all points of the rod. Let us take another point D. The rotation about C gives D the component velocity ω CD perpendicular to CD and represented by the vector DM, the end M lying on the line CE. The translation gives D the velocity ω PC, represented by the vector DH. The resultant of the velocity components, which is represented by the vector DL is perpendicular to PD and has the magnitude ω PD.

Suppose a rigid system to have a motion of rotation of angular velocity ω about a point O and at the same time a translation with a linear velocity v in a given direction. See Fig. 35. According to the law just stated, the rotation and translation can be replaced by a single rotation about a point P at a distance $r = \frac{v}{\omega}$ from O in a direction at right angles to the translation. This point P is therefore the I. C. of the system, the point that is for the instant at rest. The motion of the system is the same as if a disc of radius r fixed in the system rolled on the line MN parallel to the translation v.



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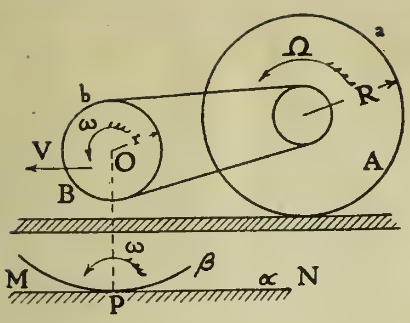
As an application, consider the motion of the sprocket wheel b of a bicycle, Fig. 36. The wheel rotates with an angular velocity ω and at the same time its center O has a velocity of translation v common to all points of the frame. Let Ω denote the angular velocity of the wheel a and R its radius; also let N and n denote,



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respectively, the number of teeth in the sprocket wheel b and in the sprocket wheel attached to a. Then we have

$$\Omega = \frac{v}{R}$$

$$\omega = \frac{n}{N}\Omega = \frac{nv}{NR}$$

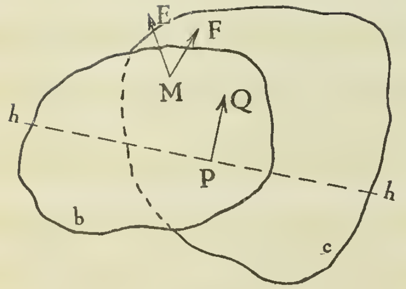
and

$$\frac{V}{\omega} = \frac{N}{n} R$$

Hence the I. C. of the sprocket wheel lies at P vertically below the point O at a distance $\frac{V}{\omega} = \frac{N}{n} R$, and the motion of the sprocket wheel can be replaced by the rolling of a disc of radius OP upon a surface MN parallel to the surface on which the bicycle is rolling.

21. Instantaneous Center of Relative Motion. So far we have discussed the plane motion of a single body or system relatively to a fixed system. We must now consider the relative motion of two moving systems and extend our conception of the instantaneous center to include this case.

Let us take two systems, b and c, Fig. 37, which have plane motion relative to a third fixed system a , which may be taken as the page on which the figure is drawn. As we know, a body having plane motion may be represented by a plane section and therefore we may conceive bodies b and c as thin discs of indefinite extent, one overlapping the other. For every point of b there is a coincident point of c directly under it. Thus at M there are two coincident points, one belonging to b and one to c. For convenience these may be denoted by M_b and M_c respectively. In general a pair of coincident points of b and c will be moving in different



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directions with different velocities. Thus, at M the vector ME gives the direction and speed of the motion of M_b and the vector MF gives the motion of M_c. But among all coincident points, there is in general one, as P, at which each point of the pair has precisely the same motion as regards both direction and speed; that is, the single vector PQ represents vector PQ represents at the same time the motion of P_b and P_c. This point P is the instantaneous center of the relative motion of b and c. In general:- The I. C. of the relative motion of two systems is the point which has the same motion whether it is considered as belonging to one system or to the other. Evidently the I. C. is the one point at which we might stick a pin through the two bodies without interfering with the instantaneous motion.

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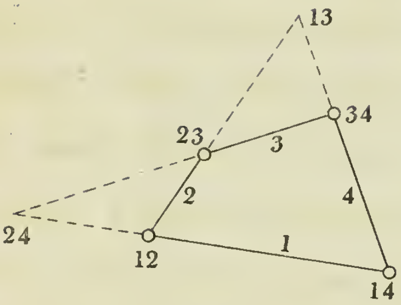
Referring again to Fig. 37, the motion of b relative to the fixed plane a is a rotation about some I. C. designated by O_{ab}. This center is the fixed point of b, that is, the point that has the same motion as its coincident point in the fixed system a. In order that point P of b shall have the direction of motion PQ, the center O_{ab} must lie somewhere on the line hh perpendicular to PQ through P. Similar reasoning shows that to give the point P of c motion in the perpendicular PQ the center O_{ac} about which c is rotating must lie in hh. Hence the three centers of the relative motions of the three bodies must lie in a straight line:- this is the "Law of Three Centers".

Text books usually denote the instantaneous centers of a series of links by O_{ab}, O_{bc}, O_{cd}, etc. For ease in writing we shall usually



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drop the O and write simply (ab), (ac), etc.:- or in case the links are numbered, we shall write (12), (13), (27), etc. Do not read these "twelve", "thirteen", "twenty-seven", but "one-two", "one-three", and "two-seven".

We shall usually number the links of a mechanism, and the fixed link we shall always call 1.

22. Application of the Law of Three Centers. Let us take the case of the ordinary 4-link chain, Fig. 38. The four links give us four systems having motions relative to one another, and there are in all six relative motions of two systems, viz. :- 2 to 1, 3 to 1, 4 to 1, 3 to 2, 4 to 2, and 4 to 3. These six motions are rotations about six different centers (12), (13), (14), (23), (24), and (34). Let us first see if any of these centers are known. The motion of link 2 relative to link 1 is a turning about the axis of the pair connecting the links; hence we have in this pair a permanent center (12). Likewise the joints between 2 and 3, 3 and 4, and 4 and 1 are permanent centers (23), (34) and (14), respectively. We have located four of the six centers as the centers of the four turning pairs; to find the remaining centers we simply use the law of three centers. Taking the three links 1, 2, and 3, the centers (12), (13), and (23), of their relative motions lie in a straight line; hence (13) lies on the line joining (14) and (34), that is the prolongation of link 2. Taking links 1, 3, and 4, the centers (13), (14), and (34), lie in a straight line, or (13) lies on the line

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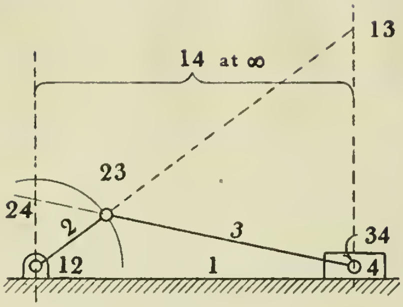

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joining (14) and (34) or the prolongation of link 4. Hence the intersection of the lines (12), (23) and (14) (34) gives us the center (13). In the same manner we find the center (24) at the intersection of the lines (12) (14) and (23) (34).

In Fig. 39 we have the steam-engine mechanism. The centers (12), (23), and (34) are readily seen to be the axes of the three turning pairs. Since the motion of 4 relative to 1 is a translation and is constrained by a sliding pair, the center (14) is at an infinite distance in a direction perpendicular to the translation. The center of the motion of the rod 3 relative to fixed link 1, the center (13), must lie on the line passing through (12) and (23), that is, on the prolongation of the crank 2; it must also lie on the line joining (34) to the infinitely distant center (14). Likewise the remaining center (24) lies at the intersection of the lines (23) (34) and a line drawn through (12) perpendicular to (12) (34). Notice that since (14) lies at an infinite distance, lines drawn from any point to (14) are parallel, and are at right angles to the direction of the motion of the slide 4.

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23. Determination of Centers. In a mechanism of n links there are $\frac{n(n - 1)}{2}$ centers some of which are known, the remainder to be found. In general, the law of three centers will give the location of the unknown centers. The general procedure is as follows:

(a) Construct a table containing all the centers. Thus:

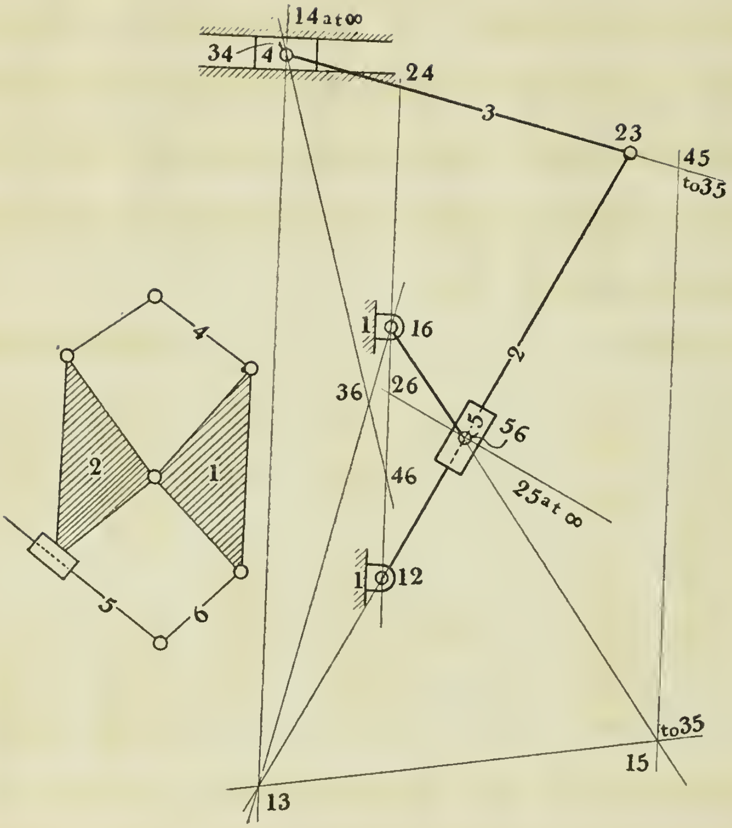
⑫	13	⑭	15	⑯	⑰	18
	㉓	24	25	26	27	28
		㉞	35	36	37	38
			㊺	46	47	㊽
				(56)	57	58
					67	68
						(78)

It will be noted that:- All the centers containing 1 are given in the upper row, those containing 2 in the second row, etc., the centers containing 8 in the last column, those containing 7 in the next to the last column, etc.

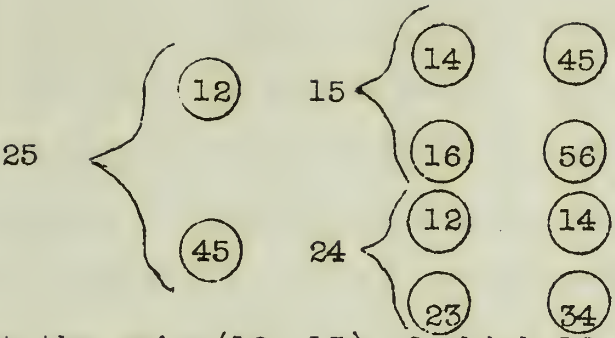
(b) Having constructed such a table, indicate known centers by drawing circles around them. The axes of all turning pairs are centers. The centers of sliding pairs are at ∞ in a direction perpendicular to the relative motion.

(c) Draw the skeleton of the mechanism in its typical form and pick out any quadric chains. As in Fig. 38, the intersection of apposite sides of quadric chains give additional known centers which might be marked in the table.

(d) Apply the law of three centers to find any center still unknown. Suppose for example, the center (25) is required, the known centers being those shown on table on page 32. From the group we select two pairs of centers as (12, 13) or (23, 35) or (27, 57) or (28, 58), the intersection of which will give 25; if one of these centers so chosen is unknown we repeat the process, and so on. A convenient form of arranging the centers is as follows:-

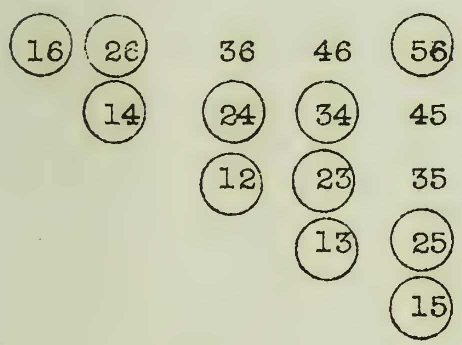

14 at ∞
to 35
25 at ∞
to 35

40



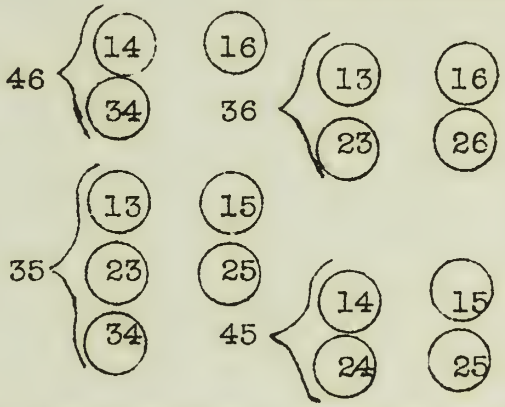
We take first the pair (12, 15) of which 12 is known and (24, 45) of which 45 is known. To determine 15 we have 14 and 45 both known, and 16 and 56 both known; likewise the pairs (12, 14) and (24, 34) determine 24. Now having (24, 45) and (12, 15) the intersection of the lines joining the centers of each pair will give us the requirod center 25.

Table of Centers



16	26	36	46	56
	14	24	34	45
		12	23	35
			13	25
				15

Combinations



The determination of the instantaneous centers of a 6-link quick-return mechanism is shown in Fig. 40. The skeleton shows that there are two quadric chains; hence in addition to the centers given by the turning and sliding pairs, we have given the four centers 13, 24, 15 and 46. The other centers are easily found. Notice that center 25 is at an infinity in a direction at right angles to the axis of the rod 2; hence a line joining 12 and 25 is drawn through 12 perpendicular to 2, one

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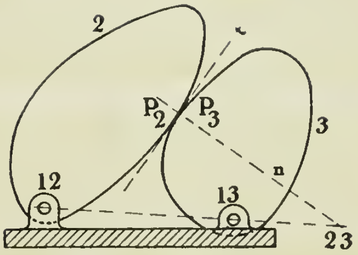

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joining 56 and 25 is drawn through the joint 56 perpendicular to 2. Likewise the line joining 13 and 14 (at ∞) is drawn through 13 perpendicular to the line of motion of the slide 4.

24. Higher Pairs. To determine all the instantaneous centers of mechanisms with higher pairs it is usually necessary to draw normals to point paths or to contact surfaces. One or two examples will illustrate this statement.

In Fig. 41 two members, 2 and 3, rotate about fixed pairs at 12 and 13, respectively, the axes of which are therefore the centers 12 and 13. Link 3 drives link 2 by direct contact, thus giving a case of higher pairing, We wish to determine the centers 23 of the relative motion of 2 and 3. Since 2 and 3 cannot cut into each other, and since they do not separate, the relative motion of the point in contact must be along the common tangent t at the point of contact P; for if this relative motion has a component along the normal, the members must either cut into each other or separate. But the relative motion of 2 to 3 is a rotation about the center 23, and evidently therefore this center must lie on a line perpendicular to the direction of the relative motion, that is, to the tangent t; hence 23 lies on the common normal drawn through P. By the law of three centers 23 also lies on the line 12 - 13; hence it lies at the intersection of the latter line with n.

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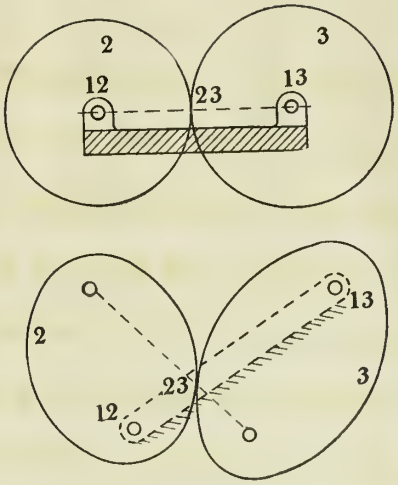
In the case of parts rolling upon each other as the pitch lines of circular or elliptical gears, Fig. 42 - the point of contact must lie on the line of centers and is itself the instantaneous



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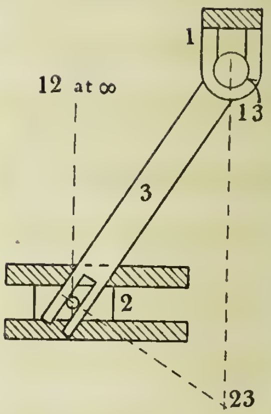

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center of the two systems.

In Fig. 43 there is a higher pair between the pin on the block

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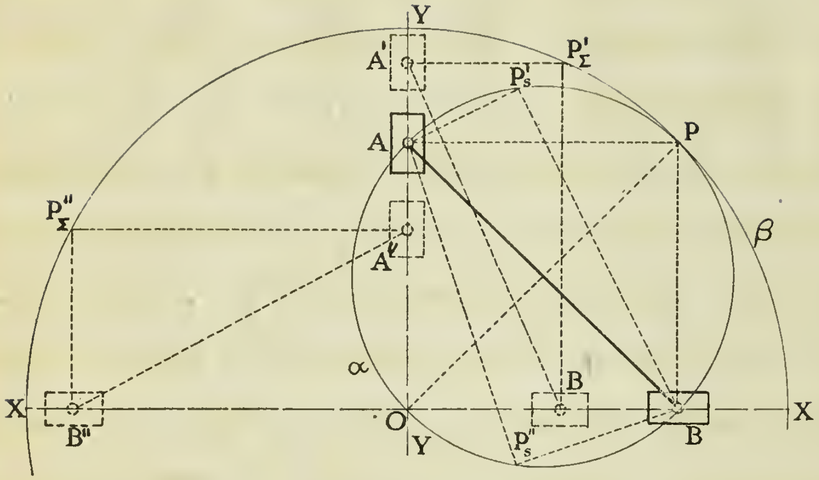
2 and the slot in the swinging arm 3. The relative motion of the pin and slot is a sliding along the slot and the center 23 lies therefore in a line through the pin perpendicular to the direction of the motion of 2 is the line joining 13 and 12 at infinity, and 23 lies also on this line; hence 23 is fully determined.

25. Centrodes. The instantaneous center P about which a system 3 is rotating relatively to a fixed system may be considered either a point of the moving system or a point in the fixed plane. Considered as a point of S we may denote it by P_s; as a point of the fixed plane or system by P_Σ. As the system S moves relative to the fixed plane Σ, the point P_s will in general occupy different positions in the moving system S; and likewise the point P_Σ will occupy different positions in the fixed system Σ. The locus of P_s in S is called the moving centrode, and the locus of P_Σ in Σ is called the fixed centrode.

As an example let us consider the motion of a rod AB the ends of which slide on the lines OX and OY. The rod belongs to the moving system S, the lines OX and OY to the fixed system Σ, which we may take as the page on which the figure is drawn.



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Erecting perpendiculars to OX and OY at B and A, respectively, we obtain the I. C.
44 of the system S at P, the intersection of the perpendiculars. When the rod reaches another position A'B' the I. C. will be at P'_Σ ; for the position A"B", the I. C. is at P''_Σ , and so on. The character of the locus of P in the fixed plane (the page) is readily seen. CAPB is a parallelogram and OP = AB; hence as AB moves, P describes a circle whose radius is the length of the rod AB, and this circle is the fixed centrode, the locus of P_Σ .

When the rod is in the position A'B' a point of the moving plane system S coincides with the I. C. P'_Σ , and when the rod is at A"B" some other point of S coincides with P''_Σ , the I. C. for the position A"B". The locus of these points of S that in turn coincide with the instantaneous centers P_Σ , P'_Σ , P''_Σ , etc., we have called the moving centrode. Taking S in some one position, say AB let us locate this curve. When the rod moves from A'B' to AB the point P'_s coincident with P'_Σ moves with the system and occupies the same position relative to AB that P'_Σ does relative to A'B'; therefore making the triangle ABP'_s equal to $A'B'P'_\Sigma$ we locate the point P'_s. In the same way if we make the triangle ABP''_s equal to $A''B''P''_\Sigma$ we obtain a point P''_s of S that will coincide with P''_Σ when AB is moved to the position A"B". From the construction of the figure the triangles $A'B'P'_\Sigma$, $A''B''P''_\Sigma$ and therefore ABP'_s, etc., are right-angled hence the points P'_s, P''_s, etc., must lie on a circle β which has AB as a diameter, and this circle is the desired

moving centrode.

For any given position of S the two centrodes have just one point in common, the instantaneous center for that position. Thus when the rod is at AB, P_s and P_Σ coincide at the point P; as A moves toward A', P_s recedes from P_Σ and P'_s approaches P'_Σ and coincides with it when A reaches A'. The relative motion of the two centrodes α and β is pure rolling. For, since the two are in contact at one point, they must either roll or slide:- but if sliding occurs P_s cannot be for the instant at rest and is not therefore the instant center.

The conslusions arrived at for the special case shown in Fig. 44 apply equally well to all cases of plane motion, and we have the following general statement:- *Any plane motion of a rigid plane system S is equivalent to and may be replaced by the rolling of two curves one attached to the moving system S and the other attached to the fixed system Σ. The point of tangency of the two curves is the instantaneous center.*

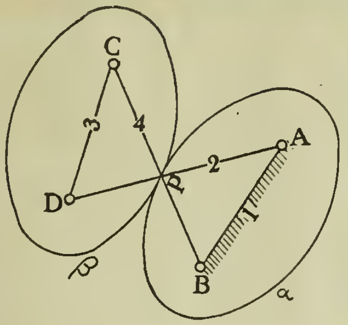
We have previously determined the centrodes for one case of plane motion. Referring to Fig. 36, it was found that the motion of the sprocket wheel b was equivalent to the rolling of a circle β upon the straight line α, the point of contact P being the I. C.; hence β was the moving centrode attached to the moving system b, and α the fixed centrode.

Suppose that we hold the system S represented by the line AB, Fig. 44, fixed, and move the system Σ represented by the lines OX and OY. It is easily seen that in this case the circle β is the fixed centrode and the circle Σ the moving centrode. This may be

shown by direct construction or it may be seen from the following considerations:- The relative motion of the system S and Σ is not changed if we introduce a motion common to both; thus considering the page as the system Σ the motion of AB relative to the page is not affected if we move the page. Suppose now that as AB moves we move the page so as to impart to S and Σ a motion just equal and opposite to the motion of AB; the result will be that S will remain at rest and Σ will be given a motion equal and opposite to the original motion of S. But since the relative motion of S and Σ is not changed its equivalent in the present case, as before, is the rolling of the centrodes α and β ; but α attached to Σ is the moving centrode and β attached to S is the fixed centrode.

An application of this principle is frequently useful in determining the moving centrode. Thus in the case of the crossed four-link mechanism, Fig. 45, we assume the link 1 fixed and attempt to find the centrodes of the motion of the link 3 relative to the link 1. Links 1 and 3 are of equal length as are also cranks 2 and 4. The I. C. of 3 for any position is evidently the point of intersection of the cranks 2 and 4. From the symmetry of the figure we have AP = CP, BP = DP, and AP + PB = AD, a constant. For all positions of P the sum of the distances of P from the joints A and B is a constant length equal to AD; hence the locus of P is an ellipse with A and B as foci; and the ellipse is the fixed centrode α . The moving centrode β is unknown; however we know that β will be the fixed centrode if we make 3 the fixed link and 1 the moving link. Under these conditions the fixed

centrode attached to 3 is a second ellipse equal to the first and having C and D as foci; when therefore 3 is the moving system the ellipse α is the required moving centrode.



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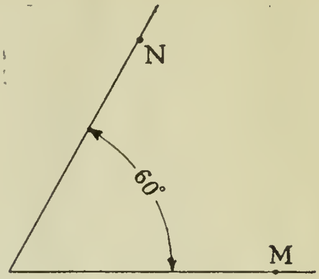

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The following problems in the construction of centrodes may be worked:--

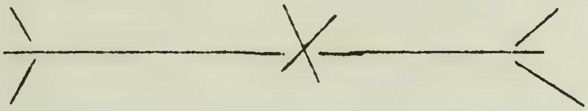
1. In Fig. 45 let 2 be the fixed link and 1 and 3 the cranks rotating about A and D, respectively; taking link 4 as the moving system S, construct the fixed and moving centrodes. Make AD = BC = 3" and AB = CD = 2".

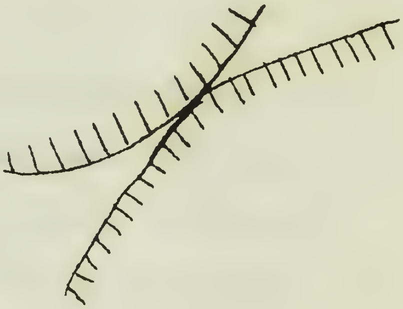
2. The moving system S consists of two lines rigidly connected and making an angle of 60° with each other. Fig. 46. The motion of the system is such that one of these lines always passes through a fixed point M, the other through a fixed point N. Construct the centrodes. Make the distance MN = 4".

3. In the slider crank mechanism, Fig. 11, take 1 as the fixed link, and 3 as the moving system S, and construct the centrodes attached to 1 and 3, respectively.

4. In the same mechanism take 4 as the fixed link and construct the centrodes attached to 4 and 2, respectively.

Draw in only the skeletons or center lines of the mechanisms. The fixed centrode is to be drawn in black, the moving centrode in blue. Corresponding points on the centrode may be denoted P_1, Q_1; P_2, Q_2; P_3, Q_3; etc., the same subscript being used for points that coincide as the centrodes roll on each other. The moving link should be shown in two or three different positions so chosen as to suggest the general character of the motion. Use full black lines for the position of reference of the moving system, dotted black lines for other positions. The construction used in finding points is to be in dotted reds. Emphasize the centrodes by section lines, as in the accompanying sketch at the right. A small sheet of transparent paper or tracing cloth will be found useful in determining points on the moving centrode.





III. VELOCITY IN PLANE MECHANISMS.

26. General Statement. In the preceding section we have studied the character of the plane motion of a rigid system without special reference to the rate of motion, that is, to the velocity. In many problems arising in machine design it is important to know the velocities of certain characteristic points of the machine parts throughout the cycle of motions of the machine. Hence it is necessary to have some general method of determining velocities from certain known or assumed conditions.

There are two general methods of attacking this problem. (1) The instantaneous center method; (2) the method of velocity polygons. Each of these methods has its peculiar advantages, and the student shoul have a thorough understanding of both.

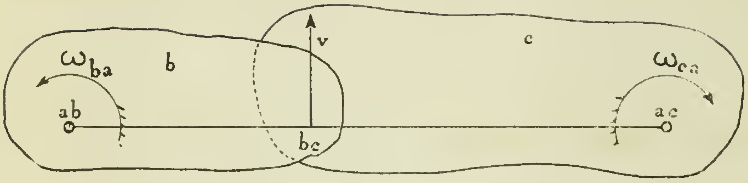
a. Method of Instantaneous Centers.

27. Relative Angular Velocities. The instantaneous centers of the relative motions of the links of a mechanism being known, it is easy to determine the relative angular velocities of various links and the linear velocities of points on the moving links. Usually the fixed link will be taken as the standard of reference.

In thus dealing with questions of velocity, the following principles are of prime importance:--

1. The instantaneous center (bc) of two links b and c is a point which has the same velocity relative to a third system "a" whether considered a point of "b" or a point of "c".

2. The linear velocities of the various points of a moving link are proportional to the distances of the points from the center



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of rotation. If the student will resolutely adhere to these principles he will have no difficulty whatever in the following developments.

Let the moving systems b and c, Fig. 47, turn about the centers (ab) and (ac), respectively, in a fixed system a; and let (bc) be the I. C. of the relative motion of b and c, (bc) being in the line joining (ab) and (ac) according to the law of three centers. Let ω_{ba} and ω_{ca} denote respectively the angular velocities of b and c about the centers (ab) and (ac), and let V denote the linear velocity of the center (bc). Considering (bc) a point of b, we have

$$\omega_{ba} = \frac{V}{(ab) - (bc)}; \qquad (1)$$

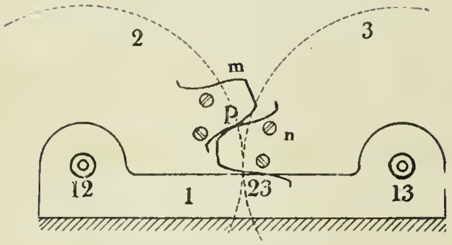
Considering (bc) as a point of c,

$$\omega_{ca} = \frac{V}{(ac) - (bc)}. \qquad (2)$$

Dividing the first equation by the second,

$$\frac{\omega_{ba}}{\omega_{ca}} = \frac{(ac) - (bc)}{(ab) - (bc)}; \qquad (3)$$

or in words, the angular velocities of b and c relative to a are inversely as the segments into which the center (bc) divides the line (ab) - (ac). This is the general law that holds for any links in a mechanism. Suppose, for example, we know the angular velocity of some link, say 2, relative to the fixed line 1, and wish to find the angular velocity of another link 5 relative to 1. We take the three centers (12), (15) and (25) - which must lie in a straight



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line - and form the proportion

$$\frac{\omega_{51}}{\omega_{21}} = \frac{(12) - (25)}{(15) - (25)}; \qquad (4)$$

whence

$$\omega_{51} = \frac{(12) - (25)}{(15) - (25)} \quad 21 \qquad (5)$$

If ω_{21} is represented by a line segment ω_{31} can be found as another line segment by easy graphical construction, the details of which are left to the student.

In the case shown in Fig. 41, in which the link 3 drives the link 2 by direct contact, we have

$$\frac{\omega_{31}}{\omega_{21}} = \frac{(12) - (23)}{(13) - (23)} \qquad (6)$$

that is, the angular velocities of the parts in contact are inversely as the segments into which the line of centers is divided by the common normal at the point of contact.

This law is important in the theory of gearing. Let the discs 2 and 3, Fig. 48, turn about the fixed points (12) and (13) and let them carry respectively, projections m and n whose curved surfaces are in contact at P at the given instant. The common normal at P cuts the line of centers in the I. C. (23) giving the segments (12) - (23) and (13) - (23). Now if (23) is a fixed point, the ratio of these segments, and as a consequence the velocity ratio transmitted, remains constant, and the motion is precisely equivalent to the rolling of two circles in contact at (23) and having their axes at (12) and (13) respectively. These circles are the pitch circles of the two wheels

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and the point of contact (23) is the pitch point. We have then the important law:-- In order that the motion transmitted by two gear teeth shall be equivalent to the rolling of the pitch circles, that is in order that the angular velocity ratio transmitted shall be constant, the common normal to the tooth curve at the point of contact must always pass through the pitch point.

Referring again to Fig. 47, it is evident that when the center (bc) lies between the centers (ab) and (ac) the rotation ω_{ba} and ω_{ca} have opposite senses; if, on the other hand (bc) cuts (ab) - (ac) externally, the rotations have the same sense.

28. Angular Velocity of Relative Motion. Having two moving systems b and c, the relative motion of the two is, as we know, a rotation about the center (bc), though this center is itself a moving point. We wish to find the angular velocity of this rotation. Suppose a to be the fixed link, (ab) and (ac) the centers in a, about which b and c are rotating, and let ω_{ba} and ω_{ca} denote respectively the angular velocities of the rotations of b and c with reference to a, the first letter of the subscript denoting the moving system and the second letter the reference system. With this rotation ω_{bc} denotes the angular velocity of system b relative to c considered as the fixed system; and vice versa ω_{cb} denotes the angular velocity of c relative to b when b is considered fixed.

Now having b and c rotating in a with angular velocities ω_{ba} and ω_{ca}, respectively, let us give the system a at the same time a rotation of angular velocity $-\omega_{ba}$; the result may be shown by the following tabular arrangement:

System	a	b	c
Angular Velocity	0	ω_{ba}	ω_{ca}
Add to each System	$-\omega_{ba}$	$-\omega_{ba}$	$-\omega_{ba}$
Result	$-\omega_{ba}$	0	$\omega_{ca} - \omega_{ba}$

The effect of giving the angular velocity $-\omega_{ba}$ to all three systems is to bring b to rest and give c the angular velocity

$$\omega_{ca} - \omega_{ba}$$

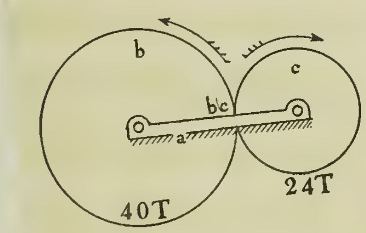
This latter quantity must be the angular velocity of c relative to b since b is at rest; or,

$$\omega_{cb} = \omega_{ca} - \omega_{ba} ,$$

and conversely

$$\omega_{bc} = \omega_{ba} - \omega_{ca} .$$

The angular velocity of the relative motion of two moving systems is the algebraic difference between the angular velocities of the two systems referred to a third system. In using this principle care must be taken to give each angular velocity its proper sign. We shall call ω positive when the sense is counter-clockwise, negative when clockwise.



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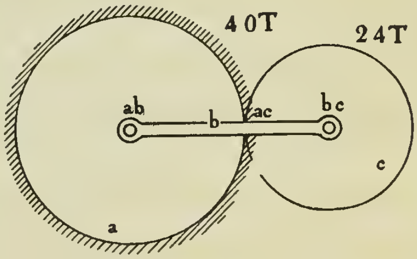

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Example 1. In Fig. 49 the moving systems are circular gears b and c with 40 and 24 teeth, respectively. If b makes 12 R. P. M. c makes 12 x $\frac{40}{24}$ = 20 R. P. M. hence, using the R. P. M. as a

unit, $\omega_{ba} = +12$ and $\omega_{ca} = -20$. The angular velocity of c relative to b, the center (bc) being the center of rotation is

$$\omega_{cb} = -20 - (+12) = -32.$$

The ratio of the angular velocities of b and c referred to a is

$$\frac{\omega_{ca}}{\omega_{ba}} = -\frac{20}{12} = -1\ 2/3.$$

an entirely different quantity. In Fig. 50 the larger gear is taken as the fixed system a. Denoting by R and r the radii of the gears, we have $\frac{R}{r} = \frac{40}{24} = \frac{5}{3}$, or $r = .6R$. Suppose the arm b to make 12 turns per minute; that is let $\omega_{ba} = 12$; to find ω_{ca} we have,

$$\frac{\omega_{ca}}{\omega_{ba}} = \frac{(bc) - (ab)}{(bc) - (ac)} = \frac{R + r}{r} = \frac{R}{r} + 1 = \frac{8}{3}$$

$$\omega_{ca} = \frac{8}{3}\omega_{ba} = \frac{8}{3} \times 12 = 32$$

For the angular velocity of the motion of c relative to b we have $\omega_{cb} = \omega_{ca} - \omega_{ba} = 32 - 12 = 20$

Example 2. In Fig. 48 let the gears be of equal radii, and rotating with an angular speed of 20 R. P. M. In the position shown the point of contact P of the tooth surfaces is .7" from the pitch point (23); required the velocity of sliding at P. We have $\omega_{21} = \frac{2\ \pi}{60} = \frac{2\ \pi\ 20}{60} = 2.0944$ radians per second; also $\omega_{31} = -2.0944$, the sense being opposed to that of ω_{21}. Then $\omega_{23} = \omega_{21} - \omega_{31} = 5.1888$ radians per second. This means that 2 is rotating about the center (23) relative to 3 with the angular velocity $\omega_{23} = 5.1888$, just as if 3 were assumed at rest. Now if 3 were at rest the velocity of the point P of 2 would be $\omega_{23} \times P - (23) = 5.1888 \times .7 = 3.622$ in. per second, and this therefore is the velocity of

sliding at P.

29. Linear Velocities. By the methods described in the preceding paragraph, the angular velocity of the motion of any link relative to any other link may be found provided the magnitude of any one angular velocity is known. Thus if ω_{21} is given, we readily determine ω_{31}, ω_{41}, etc., the angular velocities of 3 and 4, etc., relative to the fixed link 1; and from these we find ω_{24}, ω_{34}, etc. To find the linear velocity of any point of a moving link, say 4, relative to the fixed link 1, we have only to multiply the angular velocity ω_{41} by the distance of the point from the I. C. (14); for (14) is, for the instant, the point about which the system 4 is rotating, and since all points of the separate link have the same angular velocity ω_{41}, the linear velocity of the various points are proportional to their distances from (14).

Suppose a point P in the link or system 2 has a linear velocity V_p relative to the fixed link and we wish to find the linear velocity of a point Q in link 5.

The link 2 is rotating about the center (12), hence

$$\omega_{21} = \frac{V}{[P - (12)]}$$

but
$$\frac{\omega_{51}}{\omega_{21}} = \frac{[(12) - (25)]}{[(15) - (25)]}$$

$$\omega_{51} = \omega_{21}\frac{[(12) - (25)]}{[(15) - (25)]} = \frac{V}{[P - (12)]}\,\frac{[(12) - (25)]}{[(15) - (25)]}$$

Hence
$$V_q = \omega_{51}\,[Q - (15)] = V_p\frac{[Q - (15)]}{[P - (12)]}\cdot\frac{[(12) - (25)]}{[(15) - (25)]}$$

or
$$\frac{V_q}{V_p} = \frac{[Q - (15)]}{[P - (12)]}\cdot\frac{[(12) - (25)]}{[(15) - (25)]}$$

In this general formula it will be noted that the ratio of the linear velocities is the product of two fractions. The first fraction is the ratio of the distances of Q and P from the center about which they are rotating respectively, while the second fraction is the ratio of the distance of the common point (25) of the two moving links from the same centers except taken in inverse order.

Three special cases are to be noted. 1. The points P and Q may belong to the same link, say 2:- then (12) and (15) are the same point, and

$$\frac{V_q}{V_p} = \frac{Q - (12)}{P - (12)}$$

or in words, the linear velocities of the two points are proportionate to their distances from the center of rotation (12). 2. One of the centers (12) or (15) may be at an infinite distance, in which case the corresponding link has a motion of translation. If (12) lies at an infinity, we may write the equation,

$$\frac{V_q}{V_p} = \frac{Q - (15)}{(15) - (25)} \cdot \frac{(12) - (25)}{P - (12)} = \frac{Q - (15)}{(15) - (25)}$$

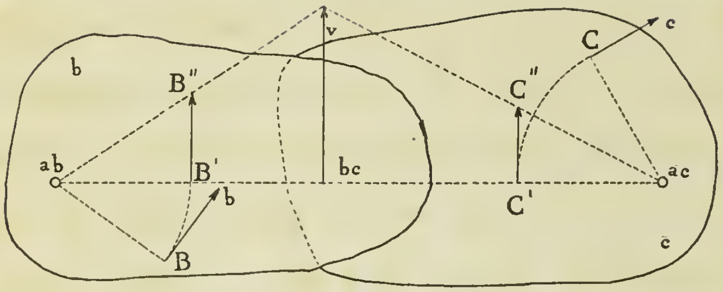
Similarly, if (15) lies at infinity we have

$$\frac{V_q}{V_p} = \frac{(12) - (25)}{P - (12)}$$

3. If the common point (25) of the two moving systems lies at infinity, we have

$$\frac{V_q}{V_p} = \frac{Q - (15)}{P - (12)}$$

When (25) is at infinity, the links 2 and 5 have evidently the same angular velocities about their respective centers of rotation, (12) and (15).



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30. Graphical Constructions for Linear Velocities. Let b and c be two moving systems and a the fixed system. Having given the velocity of a point B in system b, the velocity of a point C in system c may be found by the following easy graphical construction:

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The three centers (ab), (ac), and (bc) - which must lie on a straight line - are located. The velocity of B in system b is known and is represented by the vector BB'; the velocity of a second point of B may therefore be found. For this second point we choose the center (bc). Revolving B about (ab) as a center, we obtain B, in the line (ab) - (bc) the velocity of which is B_1B'' = BB'. Drawing a line through (ab) and B" and a vector through (bc) perpendicular to (ab) - (bc), we find the intersection V. Then the vector (bc) - V represents the velocity of the point (bc) of system b:- for from similar triangles, (bc) - V : BB" = (ab) - (bc) : (ab) - B. But (bc) has the same velocity in system c; hence we know the velocity of one point (bc) of system c and we can readily find the velocity of a second point C. Remembering that C is turning around (ac) as a center we draw the line (ac) - V, revolve C to C_1 in the line (ac) - (bc) and through C_1 draw a vector perpendicular to (ac) - (bc) cutting (ac) - V in C". From the similar triangles (ac)C_1C'' and (ac) (bc) V we have

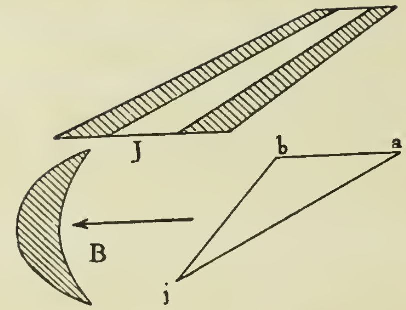
C_1C'' : (bc) - V ÷ (ac) - C_1 : (ac) - (bc) ;

hence since (bc) - V represents the velocity of (bc), C_1C'' must represent the velocity of C_1. Since C and C_1 are at the same distance from the center of rotation (ac), C has a velocity equal in magnitude

to that of C_1; hence drawing CC' = C_1C'' and perpendicular to (ac) - C, we hace in CC' the required velocity of C.

It should be noted carefully that *we pass from one moving system to another by means of the instant center of the two systems, the point common to both*. Suppose for example we take link 1 of a machine as the fixed system and that the velocity of a point on 2 is known. By the triangle construction of Fig. 51, we can find the linear velocity of any points whatever of system 2. Let us find therefore the velocities of the points (23), (24), (25), and so on. Now (23) in system 3 has the same velocity as (23) in system 2, likewise (25) in 5 has the same velocity as (25) in system 2; hence we know now the linear velocity of one point in each moving link. To find the velocity of a second point of system 5 we know that the point (15) is the center about which 5 is turning and the velocities of various points are proportional to their distances from (15); therefore knowing the velocity of (25) and its distance from (15) we can find the velocity of a second point by the usual triangle construction.

A special case of importance is that in which one link has a motion of translation with respect to the fixed link. In this case all points of the moving link have the same velocity. Thus, in Fig. 40 link 4 has such a motion and the points (34), (24), (45), and (46) have the same velocity. If we know the angular velocity of the crank 6 to be ω then ω (16) - (46) is the linear velocity of (46) [(46) being a point of 6 as well as of 4] and is therefore the linear velocity of the slide 4. Since (46) must always be on the vertical (14) - (16), the velocity of the slide will vary with the



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distance of (46) above or below (16).

b. Method of Velocity Polygons.

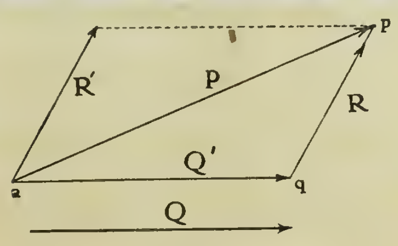
31. General Principle. Two bodies, each of which is moving relative to some fixed body have in general relative motion. Thus the crank and connecting-rod of an engine have each a certain motion relative to the bed and they have also a motion relative to each other, which is a turning about the axis of the crank pin.

In studying the relative motion of bodies the following principle is of great inportance:- The relative motion of two bodies is not affected by any motion they may have in common. For example, the relative motion of the crank and connecting-rod is the same whether the engine is stationary or moving. In a marine engine the relative motion of the parts is unaffected by the rolling of the ship.

From this principle it follows that in studying the relative motion of two bodies, we may neglect any motion common to both; also we may give them a common motion without affecting their relative motion.

32. Relative Velocity. The velocity of a point P relative to a second point Q is defined as the velocity P would have if Q were at rest. The two points may belong to different moving systems or to the same system.

Let the vectors P and Q, Fig. 52, represent respectively, the



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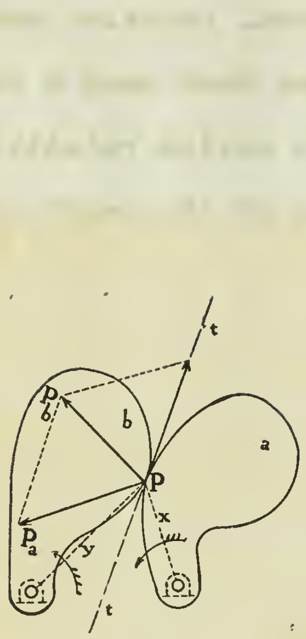

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velocities of two points P and Q in some fixed system A. Drawing Q' from one extremity of P, and completing a parallelogram, we see that P is the resultant or geometric sum of Q' and R'. The relative motion of P and Q is unchanged if we subtract the velocity Q' which they have in common. This subtraction brings Q at rest and g gives P the velocity R' = R; hence R represents the velocity of P relative to Q. An easy way of getting the relative velocity is as follows:- from a point a, Fig. 52, lay off ap = P's velocity relative to A, and aq = Q's velocity relative to A. Draw the closing side pq.

Then qp = P's velocity relative to Q

or pq = Q's velocity relative to P.

As an example of the application of this important principle, take the action of a steam jet against a moving bucket B. If the fixed system is A, lay off aj to represent the velocity of the bucket. Then bj gives the velocity of the jet relative to the bucket.

35. Two Special Cases. An important application of the principle of relative velocity is found in mechanisms in which motion is transmitted by direct contact. Thus in Fig. 55 the bodies a and b turn on the pins X and Y respectively, and are in contact at P. Of the two points in contact let P_a denote that belonging to a and P_b that belonging to b. Evidently the velocity of P_a is perpendicular to XP, that of P_b, perpendicular to YP. Since P_a and P_b do not have the same velocity, they must have a certain relative velocity, which can be determined if its direction is known. It is clear that this direction must be that of the common tangent tt at the point of

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[illegible]

[illegible]

[illegible]

[illegible]

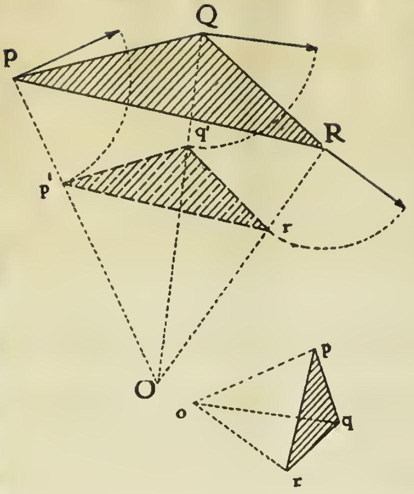
[illegible]

contact, for if there were a component perpendicular to the tangent the bodies would separate or cut into each other.

Suppose P_{pa} the velocity of P_a be known. Resolve P_{pa} into components one P_{pb} perpendicular to YP and the other along it. Then P_{pb} is the velocity of P_b and $p_b p_a$ is the velocity of P_a relative to P_b.

In all such problems we have usually the directions of the velocities of the coincident points relative to the fixed or reference body, and the direction of their relative velocity is given by the conditions of the problem. With the three directions and the velocity of one point, known, the construction of a triangle gives the other velocities.

Suppose the two points P and Q whose relative velocity is required belong to the same rigid system. The velocity of P relative to Q has been defined as the velocity of P when Q is brought to rest. Now if the two points belong to the same system the only plane motion of the system possible with Q at rest is a rotation about Q as an instantaneous center, and as a result of this motion P must have a volosity at right angles to the virtual radius PQ. Hence *the relative velocity of two points of a rigid system is perpendicular to the line joining the two points*. The truth of this principle is evident from other considerations. If the relative velocity of P to Q or of Q to P has a component along the line PQ the points must approach each other or recede from each other, but P and Q are points of a rigid system and therefore must remain at a fixed distance from each other; hence the relative velocity can have no component along PQ and must remain perpendicular to PQ.



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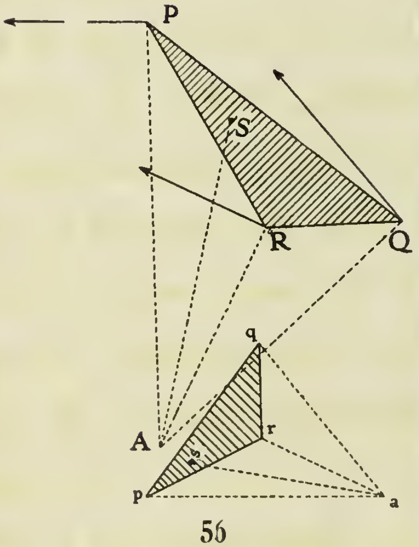

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As a direct consequence of the principle just stated we have the following:- *The components of the velocities of two points P and Q of a rigid system along the joining line PQ are equal.*

34. *Velocity Images.* Let P, Q and R, Fig. 56, be three points of a rigid system, and suppose the velocities of the points P and Q

56 57

to be known. It is required to find the velocity of the point R, which may be any point of the system. From any point a we draw ap and aq equal to and parallel to the velocities of P and Q, respectively. By the preceding paragraph, vector qp represents the velocity P relative to Q and is therefore perpendicular to QP. We note therefore that the ends p and q of the vectors ap and aq lie upon a line perpendicular to PQ. For the same reason the end of the vector ap and the end of the vector representing the velocity of the point R will lie upon a line perpendicular to PR, and the end of aq and of this last vector will lie upon a line perpendicular to QR. Hence, through p we draw a line perpendicular to PR and through q a line perpendicular to QR the intersection of these lines is the end of the vector pr that represents the velocity of the point Q.

By construction the sides of the triangle pqr are perpendicular to the sides of the triangle PQR; hence the two triangles are similar

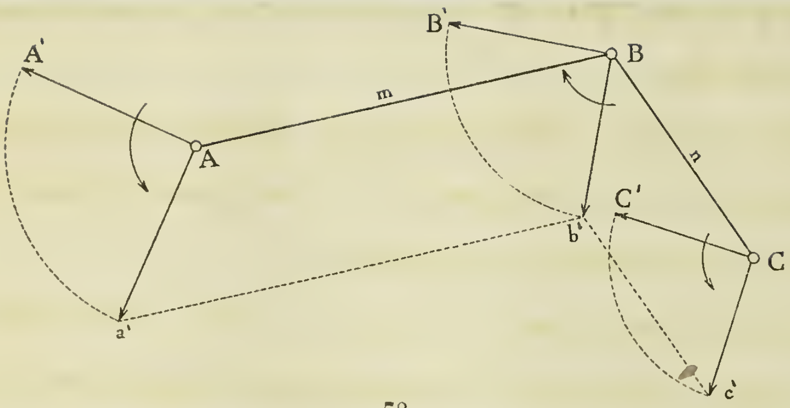
and we have the important proposition:- *If from any point as a pole the vectors representing the velocities of various points of a system are laid off, the ends of these vectors form a system similar to the original system - i.e. an image of the original system.*

To find the velocity of any point of the system, we locate the corresponding point on the image and connect it with the pole O; thus to find the velocity of the point S, we locate s in the same position relative to triangle pqr that S occupies relative to triangle PQR and draw as. Evidently the point a of the image corresponds to the point of the original system that has no velocity, hence, making the triangle APQ similar to the triangle apq, we locate the point A of the original system that is at rest and which is therefore the instantaneous center of the system.

35. *Resolved Velocities.* For the sake of convenience in drawing it is usually advisable to revolve all velocities through a right angle. When this is done the lines of the velocity image are parallel to the corresponding lines of the original system. Thus in Fig. 57 the velocity vectors at P and Q are revolved clockwise through 90° to Pp' and Qq' respectively. Evidently O, the intersection of Pp' and Qq' (prolonged), is the I. C. of the system. Hence from the proposition

$$OP : OQ = Pp' : Qq'$$

it follows that p'q' is parallel to PQ. For the same reason p'r' must be parallel to PR and q'r' to QR. That is, the triangle p'q'r' formed by the ends of the revolved velocity vectors is similar to and has its sides parallel to the triangle PQR. Likewise if the



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revolved vectors are laid off from a pole O, the ends lie in a triangle pqr which has its sides parallel to PQR.

36. Velocity Polygons. It has been shown that the ends of the velocities of the points of a moving system form an image of the system, the corresponding lines of the system and the image being perpendicular. If, however, the velocicities are revolved through a 90°, the corresponding sides of the system and the image are parallel. This principle may easily be extended to mechanisms with severa several moving systems.

Before taking up the construction of images for complicated mechanisms, however, it is necessary to show some geometrical constructions that are frequently required.

1. Two moving links m and n, Fig. 58, have the common joint B. The ends A and B have velocities represented respectively by the vectors AA' and CC'. Required the velocity of the joint B. We revolve AA' and CC' through 90° to Aa' and Cc', then through a' and c' draw lines parallel, respectively, to AB and CE giving the intersection b'. We know that the end of B's revolved velocity must lie somewhere on the parallel through a' (B being a point of m), and since B is also a point of n it must lie somewhere on the parallel through c'; hence it lies at b' and Bb' is the revolved velocity of the point B. Turning Bb' back through 90° we get BB' as the actual velocity.

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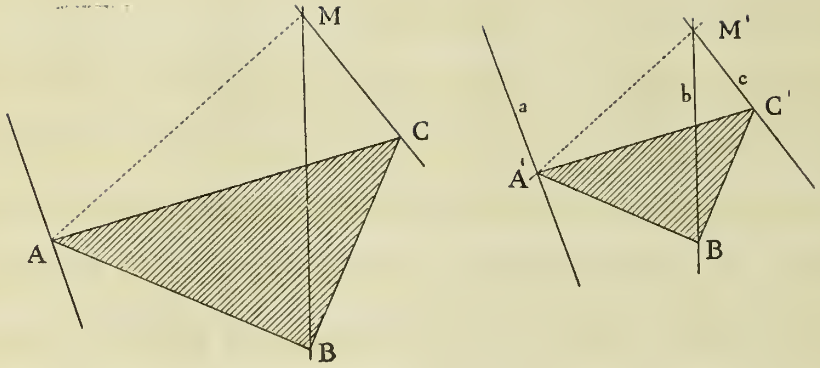
2. A parallel image of a triangle ABC, Fig. 59, must have its



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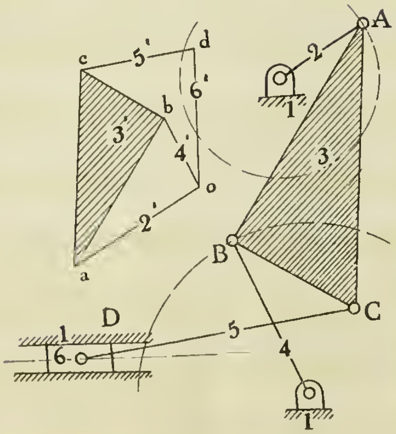

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vertices A', B', and C' upon three given lines, a, b, and c, respectively. To construct the image. Through B and C draw lines parallel to b and c intersecting in M and join M to A. Through M', the intersection of b and c, draw M'A' parallel to MA. A' is then the one vertex of the required image B' and C' are found by drawing A'B' parallel respectively to AB and AC. The proof of this construction is left to student.

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A velocity polygon is a figure in which all the velocity images of the moving links are concentrated and the lines of which represent the actual or revolved velocities of various points of the links. The construction of the velocity polygon is readily effected by the principles heretofore given. A few examples will be used as illustrations.

In Fig. 60 is shown in skeleton form the mechanism of the Atkinson gas-engine. Starting with the crank pin A, we know that A's velocity is perpendicular to the crank and its revolved velocity lies along the crank. Hence draw Oa to represent A's revolved velocity relative to the fixed link. Going not to joint B we know that B's revolved velocity must be along the link 4; hence, through O draw an indefinite line parallel to link 4. B's velocity relative to A must be perpendicular to AB, hence, the revolved relative velocity must be parallel to AB. Through a therefore draw line parallel to AB and thus obtain the intersection b. Note

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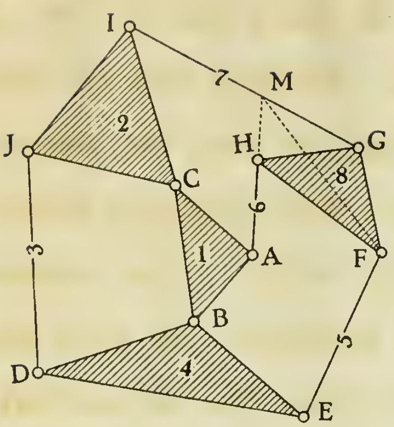

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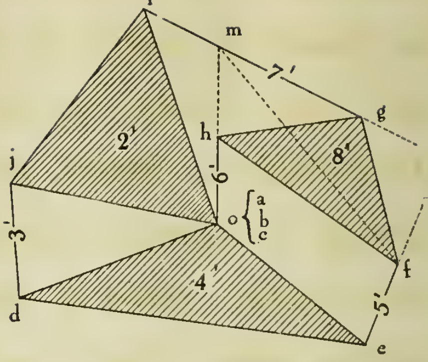

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that ab is the parallel image of AB. To get c, draw through a and b parallels to AC and BC respectively. We have now 2', 3' and 4' as the parallel images of links 2, 3 and 4. Passing now to link 5 we note that we have the image of point C and we know the direction of D's velocity. Hence, draw Od parallel to D's revolved velocity and through c, already found, draw a parallel to CD. The intersection of these lines gives d, and completes the polygon. In this polygon the vectors drawn from O, as Oa, Ob, Oc, Od gives the velocities of the corresponding points relative to the fixed link, while the sides of the images give relative velocities. Thus:--

ab = B's velocity relative to A (revolved)
dc = C's " " " D "
bd = D's " " " B "
etc.

The velocity polygon of the 8-link chain of Fig. 61 is shown in

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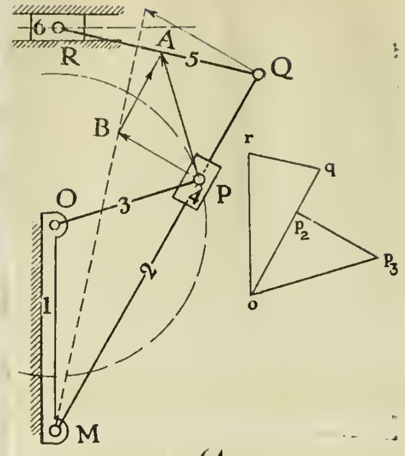
Fig. 62, the link 1 being fixed. The angular velocity ω_{41} of link 4 is assumed to be known and the size of the image 4' of 4 is thus determined. If $\omega_{41} = 1$, the image has the same dimensions as the system; as drawn $\omega_{41} = \frac{4}{3}$, hence the sides of 4' are $\frac{4}{3}$ of the corresponding sides of 4. Through 34' the image 3' is drawn parallel to link 3 and from O the image O-23' parallel to 12 23, the

intersection giving 23'. The image is readily drawn, 0 - 27' and 23' - 27' being respectively parallel to 12 - 27 and 23 - 27.

To locate the image 8' is more difficult. Through 27' we draw the line 7' parallel to 7 and through 45' the line 5' parallel to 5. Also we know that the revolved velocity of 68 must lie along 6 and that the image 6' will be parallel to 6 and drawn from 0; hence we have three lines, 5', 6', and 7' on which lie the three vertices 58', 68', and 78' of the image 8', the sides of which must be parallel to the sides of link 8. We prolong 6 to intersect 5 in M and likewise 6' to intersect 5' in M'; then drawing through M' a line parallel to M - 78, we obtain 78' as the intersection of this line with 7'. Having one vertex 78' the image is easily constructed.

Exercise. - Draw the velocity polygon when 4 is taken as the fixed link. The angular velocity of link 3 is known.

37. Velocity Polygons for Special Cases. Many mechanisms have sliding pairs between moving links. Take for example the quick-return shaper mechanism, shown in Fig. 40 in which links 2 and 5 have a sliding pair. The block 5 is added merely to give lower pairing, and so far as the motion is concerned we may omit the block entirely and use a single higher pair between 2 and 6 as shown in Fig. 64. The pin P on link 6 moves in the slot in link 2 and the driving is by direct contact. The principles of Art. 33 may be used in finding the velocities. Thus, let PA represent the velocity of the pin p, and construct a triangle with sides PB and BA perpendicular to and parallel to MQ, respectively; then PB represents the velocity of the point of 2 coincident with P, and BA is the velocity of sliding.



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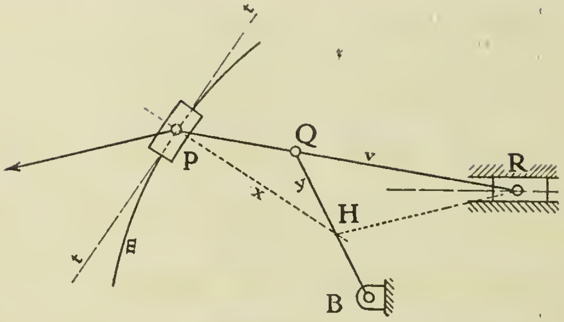

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From O, Fig. 64, lay off Op_3 velocity of P and draw through O a line perpendicular to the axis of 2; then through p_3 draw a

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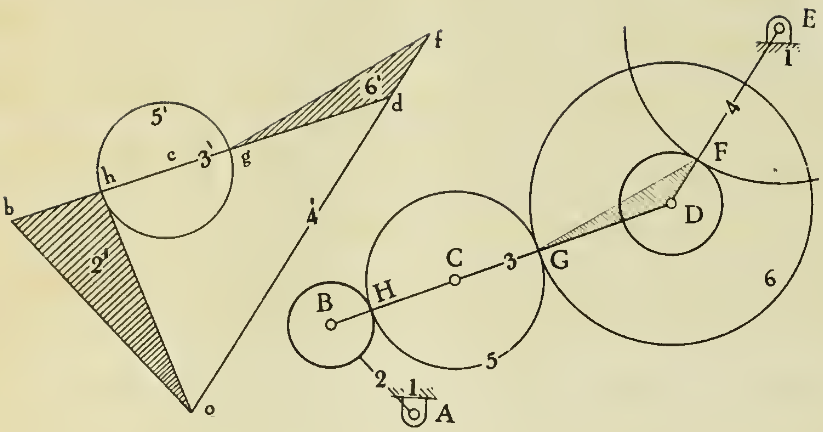
line parallel to 2, thus giving the point p_2, Op_2 gives the velocity of the point of 2 coincident with P. Now locate q so that O, p_2, and q have the same relative position as O, P, and Q. Then Oq represents the velocity of the end Q of rod 2.

As difficult a problem as is likely to be found in practice is shown in Fig. 65. A link m is given a complex motion by means of two eccentric rods and a block sliding on this link is attached at P to a valve rod v. At Q the rod v is pinned to a radius rod QB. Let us assume that the velocity of the point P of the arc m is known; the problem is to find the velocity of joint R which is the same as that of the valve. From O, Fig. 66, lay off Op_m to represent the revolved velocity of P (of the arc m). The velocity of P relative to m has the direction of the tangent tt; hence through P_m draw xx perpendicular to t, and somewhere on xx will be a point p giving a triangle Op_mp, of which side Op represents the revolved velocity of P (as a point of v) and side p_mp represents the velocity of sliding.

Through O draw also an indefinite line Oy parallel to the radius rod BQ. Then somewhere in this line is a point q such that Oq gives the revolved velocity of Q. Likewise, through O draw OZ in



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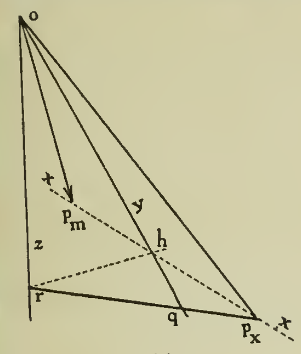
the direction of the revolved velocity of R, and on OZ must lie a point r such that Or represents the revolved velocity of R. By the theory of velocity images, the three points p, q, and r lying respectively on lines xx, Oy, and OZ are images of the points P, Q, and R and must lie in a straight line parallel to rod v. Through P draw a line x' cutting QB in H and join H to R. From the intersection h of xx and Oy, Fig. 66, draw hr parallel to HR and through r draw a line parallel to rod v. This line is the required image. Vector Or represents the velocity of the valve, and vector $p_m p$ the velocity of slipping between the block and the link.



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Study this construction very carefully. See if you can obtain the same result by finding the instant center of rod v.

A final example in the construction of velocity polygons is

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shown in Figs. 67 and 68. The mechanism consists of a four-link chain on which is superimposed a gear train. The first wheel is rigidly fixed to crank 2 and turns with it. The gear and pinion 6, are fixed together and form one link; the gear engages with the idle wheel 5, the pinion with the wheel 7.

The angular velocity of crank 2 is constant and is assumed to be known, and the angular velocity of a point on the circumference, is required.

From O, Fig. 67 lay off Ob to represent the assumed revolved velocity of point B. Likewise through O draw a parallel to DE and through b a parallel to BD. The point d is thus located.

Now on bd locate c the image of joint O on the link 3. Point H is a point of 2. To locate its image construct the triangle ABH and in Fig. 68 make the similar triangle Obh. Observe that h does not belong to the image bd of the coupler 3. Now direct attention to gear wheel 5. The image of the center O has been found, also the image of the point of contact H with wheel 2. Hence with oh as a radius draw a circle and thus locate g opposite to h. Turning now to the gear and pinion 6, we have the images of points D and G; hence to get that of F construct triangle DGF and in the polygon draw through 8 a parallel to GF. The point f is thus found. Vector Of represents the velocity of F, a point on the circumference of gear 7.

Note that $\omega_{71} = \frac{Of}{EF}$, and $\omega_{41} = \frac{Od}{ED}$;

hence, $\omega_{71} > \omega_{41}$.

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